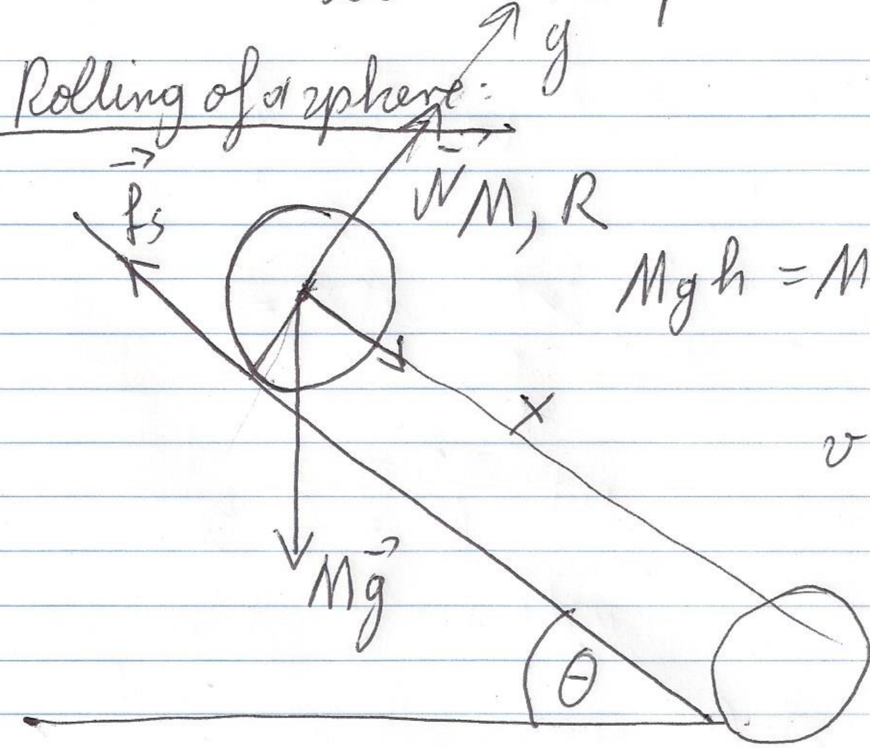
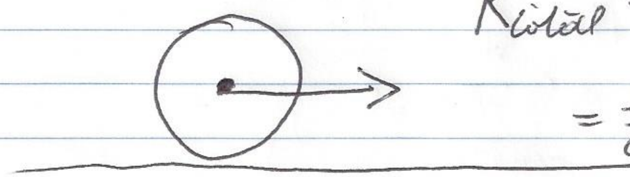
Rolling of a sphere:



$$Mgh = Mg\,x \cdot \sin\theta$$

$$v^2 = 2ax$$

$$\frac{7}{10}Mv^2 = K_{tot}$$



$$K_{total} = K_{rot} + K_{linear}$$
$$= \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv^2$$
$$= \frac{1}{2}\frac{2}{5}Mr^2\omega^2 + \frac{1}{2}Mv^2$$
$$= \frac{1}{5}Mv^2 + \frac{1}{2}Mv^2$$
$$= \frac{7}{10}Mv^2$$

$v \rightarrow \omega r$

$M \rightarrow \underline{I}$

$\mu_s = ?$ linear motion

$$\vec{f}_s + \vec{N} + m\vec{g} = m\vec{a}$$

x: $-f_s + mg\sin\theta = ma$

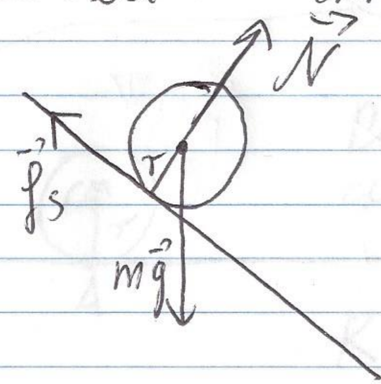
y: $N - mg\cos\theta = 0$

$$f_s = \mu_s \cdot N = \mu_s \cdot mg\cos\theta$$

$$f_s = \mu_s mg\cos\theta$$

$$-\mu_s \cancel{m} g\cos\theta + \cancel{m} g\sin\theta = \cancel{m} a$$

$$\sum \tau_{ext} = I_{CM}\,\alpha$$



$$f_s \cdot r = I_{CM} \cdot \alpha$$

$$= \frac{2}{5} m r^2 \cdot \alpha$$

$$r\alpha = a$$

$$f_s \cdot \cancel{r} = \frac{2}{5} m \cancel{r}\, a$$

$$\mu_s \cancel{m} g\cos\theta = \frac{2}{5} \cancel{m} a$$

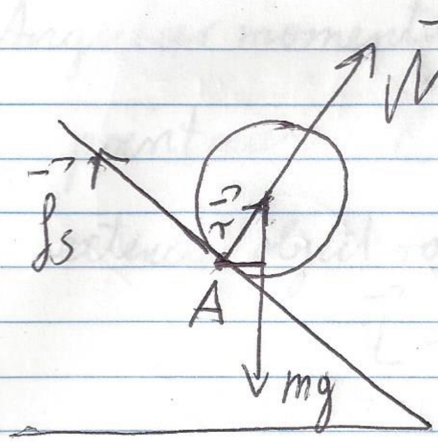
$$x:\ -f_s + mg\sin\theta = ma$$

$$-\frac{2}{5}\cancel{m} a + \cancel{m} g\sin\theta = \cancel{m} a$$

$$g\sin\theta = \frac{7}{5} a \ ; \ a = \frac{5}{7} g\sin\theta$$

$$\mu_s \cancel{g}\cos\theta = \frac{2}{5} \cdot \frac{5}{7} \cancel{g}\sin\theta$$

$$\mu_s = \frac{2}{7}\tan\theta$$

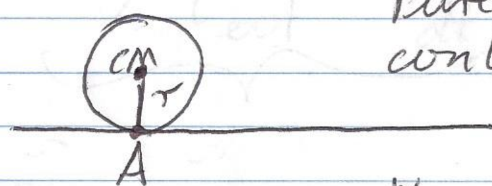


Consider torques around A

Only $\cancel{\vec{f_s}}$ $m\vec{g}$ creates a torque around A.

$$|\vec{r} \times m\vec{g}| = rmg\sin\theta$$

Pure rotation around the contact point A.



$$K = \frac{1}{2} I_A \omega^2$$

$$= \frac{1}{2}\left(I_{cm} + mr^2\right)\omega^2$$

$$= \frac{1}{2}\left(\frac{2}{5}mr^2 + mr^2\right)\omega^2$$

$$\tau_A = I_A \cdot \alpha = rmg\sin\theta$$

$$\frac{7}{5}\cancel{m}\underbrace{r^{\cancel{2}} \cdot \alpha}_{a} = \cancel{r}\cancel{m}g\sin\theta$$

$$a = \frac{5}{7} g\sin\theta$$

Angular momentum :

point mass $\vec{l} = \vec{r} \times \vec{p}$

extended object rotating around a point A

$\vec{L} = I_A \vec{\omega}$ fixed axis rotation.

$$\underbrace{\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}} \; ; \quad \sum \vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

$\Downarrow$

linear momentum cons.

$\sum \vec{p}_i = \sum \vec{p}_f$

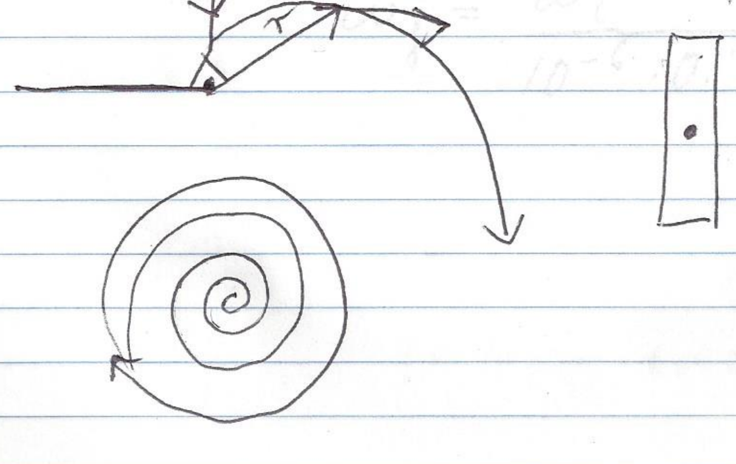
if $\sum \vec{\tau}_{ext} = \vec{0}$

$$\frac{d\vec{L}}{dt} = 0$$

$$\boxed{\vec{L}_f = \vec{L}_i \; ; \; I_1 \omega_1 = I_2 \omega_2}$$

angular momentum conservation

$I \approx m r^2$ $\quad L = I \omega$



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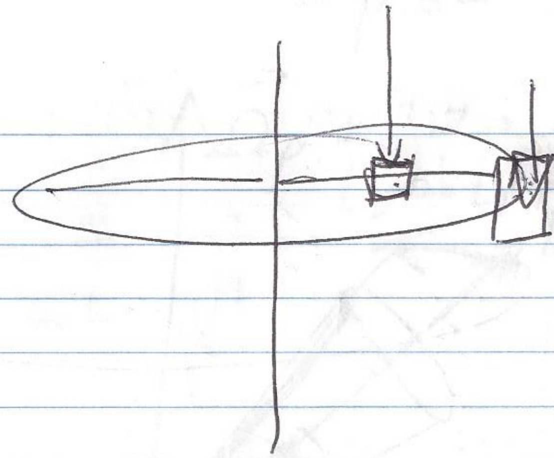
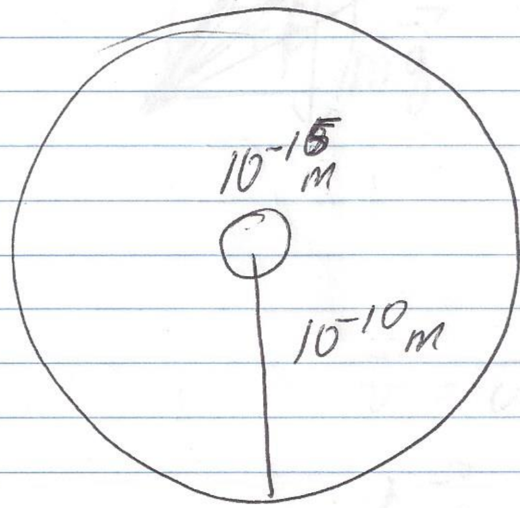


$L_i = L_f = I\omega$



star spinning originally 3 revolutions per 100 years

$L_i = I_i \omega_i$

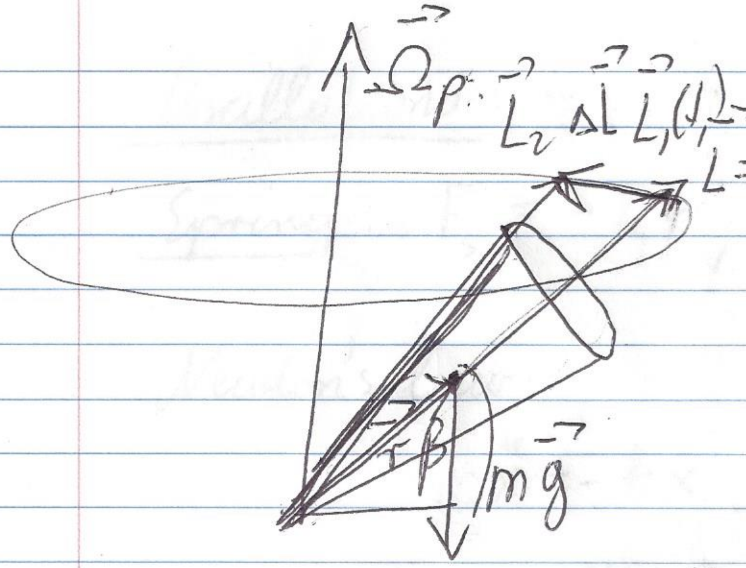
$= m_i r_i^2 \omega_i$

$$L_f = m_f \cdot r_f^2 \cdot \omega_f$$

$$\cancel{m_i} \cancel{r_i^2} \omega_i = 0.5 \cancel{m_i} \cdot 10^{-6} \cancel{r_i^2} \omega_f$$

$$\omega_f = \frac{\omega_i}{10^{-6} \cdot 0.5} = 2 \cdot 10^6 \cdot \omega_i$$

Spinning top.

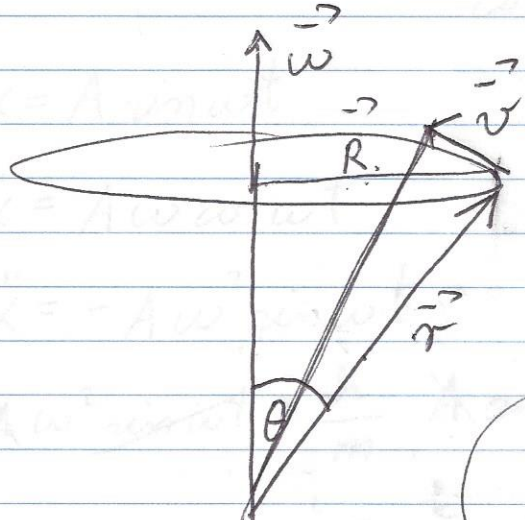


$\vec{L}=I\vec{\omega}$ top spins around its central axis with ω.

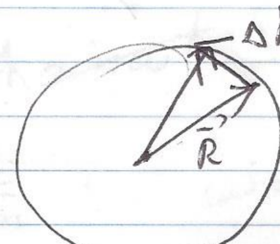
$$\vec{\tau}=\vec{r}\times m\vec{g}$$

$$\vec{\tau}=\frac{d\vec{L}}{dt}=\vec{\Omega}_p\times\vec{L}$$

$$\vec{v}=\vec{\omega}\times\vec{r}=\frac{d\vec{r}}{dt}$$



$$|\vec{R}|=r\sin\theta$$



$$\left|\frac{\Delta\vec{R}}{\Delta t}\right|=R\cdot\frac{\Delta\theta}{\Delta t}$$

$$\Omega_p\cdot L\cancel{\sin\beta}=r\cdot mg\cancel{\sin\beta}$$

$$v=r\sin\theta\frac{d\theta}{dt}$$

$$\Omega_p=\frac{rmg}{L}$$

$$\vec{v}=\vec{\omega}\times\vec{r}$$

$$L=I\omega$$

Oscillations:

Spring: $F_s = -kx$; $U = \frac{1}{2}kx^2$

Newton's law: $\dot{x} = \frac{dx}{dt}$ $\ddot{x} = \frac{d^2x}{dt^2}$

$$m\ddot{x} = -kx$$

$$x(t)$$

$$\ddot{x} + \frac{k}{m}x = 0$$ linear differential equation of order 2.

$$\cos\ddot{x} + \frac{k}{m}\ln x = 0$$

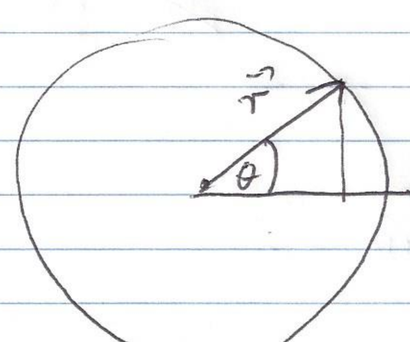
$$x = A\sin\omega t$$ $$y = \sin kx$$

$$\dot{x} = A\omega\cos\omega t$$

$$\ddot{x} = -A\omega^2\sin\omega t$$

$$-\cancel{A}\omega^2\cancel{\sin\omega t} + \frac{k}{m}\cancel{A}\cancel{\sin\omega t} = 0$$

$$\omega^2 = \frac{k}{m} \qquad \omega = \frac{2\pi}{T}$$



$$\theta = \omega t$$

$$x = r\cdot\cos\omega t$$

$$y = r\cdot\sin\omega t$$