1. [10] Find the four solutions of the complex equation $\hat{z}^4 = \sqrt{2} + i\sqrt{3}$. Calculate the second solution (k=1) in the form of a + ib.

$r = \sqrt{5}\,; \theta = 0.886 = A\tan\sqrt{\frac{3}{2}}$

$\hat{z}_n = 5^{0.125} e^{i\left(\frac{0.886+2k\pi}{4}\right)}; n = 0,1,2,3$

$\hat{z}_1 = 1.2228 \cdot (\cos 1.7922 + i \sin 1.7922) = -2.687 + i1.220$

2. The position of a particle is given by $x = 4m\cos(3\pi t + \pi)$ where x is in m and t is in s.

3. [5] A simple pendulum of length 2.68 m is released from an angle of 11.0 degrees with the vertical. **Derive** the differential equation for the pendulum from Newton's laws and find its period and angular frequency. (Show the forces in the drawing !!! Hint: $a_\theta = r\alpha$)

$-mg\sin\theta = l\ddot{\theta} \Rightarrow \ddot{s} + \frac{g}{l}s = 0$ for small angles

$\omega = 1.91/s; T = \frac{2\pi}{\omega}$

[5]Find the speed and acceleration of the pendulum when it makes a 6 degree angle with the vertical.

$\dot{s}(\text{at } 6°) = -0.928 m/s$

$\ddot{s} = -1.02 m/s^2$

4. A certain oscillation is described by the differential equation: $m\ddot{x} + kx = 0$
m=1.5kg; k=36.5N/m.
[10] If this oscillation has an amplitude of 12 cm, find its kinetic energy and its potential energy at the time t=T/5 where T is its period. Find ω and T first.

ω=4.93/s; T=2π/ω

KE=0.238J; U=0.025J

5. Write down the differential equation for a spring with mass m, spring constant k, and damping factor b, performing damped oscillations.
Show that the function $x = Ae^{i\omega t}$ satisfies this equation provided that ω satisfies a certain equation. Find ω and write down the general solution x.

6. A spring with mass 1.00 kg, k=25N/m undergoes a damped oscillation according to:

$x = 2.44e^{-\frac{b}{2m}t}\cos\omega_1 t$ in cm.

The maximum amplitude of this oscillation becomes 1.00 cm after 125 seconds. Determine b.

7. A meter stick of mass 180g is suspended at the 80cm mark in such a way that it can freely rotate around this axis in a vertical plane. It is released from an initial angle of 14 degrees. Assume that it performs simple harmonic motion and find its period of oscillation.

T=1.524 s.

8. Find the exponential representation of the complex number:

$\frac{1}{4+3i}$

Solution: r=$5^{0.5}$, θ=Arctan0.75=0.6435; $\hat{z} = \sqrt{5}e^{-i(0.6435+2n\pi)}$

9. A damped oscillating spring with k=40.0N/m, m=1.00kg, b=0.125 is driven by an exterior oscillating force of 5.00N and an angular frequency of 6.25rad/s. Derive the amplitude of the oscillation after the spring has adjusted to the exterior frequency. Use complex functions for this derivation. Calculate the amplitude.

$$\left(-\omega_f^2 + i\frac{b\omega_f}{m} + \omega_0^2\right)\hat{A}e^{i\omega_f t} = \frac{F}{m}e^{i\omega_f t}$$

$$\hat{A} = \frac{F/m}{\omega_0^2 - \omega_f^2 + i\frac{b\omega_f}{m}} = \frac{F/m}{\underbrace{\sqrt{\left(\omega_0^2 - \omega_f^2\right)^2 + \left(\frac{b\omega_f}{m}\right)^2}}_{r}}e^{-i\theta};\ with\ \theta = \tan^{-1}\frac{\frac{b\omega_f}{m}}{\omega_0^2 - \omega_f^2}$$

$$\left|\hat{A}\right| = \frac{5}{\sqrt{\left(40 - 39.063\right)^2 + \left(\frac{0.125\cdot 6.25}{1}\right)^2}} = 4.097m$$

We can move the angle into the argument of the sine or cosine function as a phase shift.

Note that the absolute values of a complex number is given by:

$$\left|\hat{z}\right| = \sqrt{\hat{z}\cdot\hat{z}^*} = \sqrt{(a+ib)(a-ib)} = \sqrt{\left(a^2+b^2\right)} = \sqrt{re^{i\theta}\cdot re^{-i\theta}} =$$

$$= \sqrt{r\left(\cos\theta + i\sin\theta\right)\cdot r\left(\cos\theta - i\sin\theta\right)} = r$$

The expression $\hat{z}^* = re^{-i\theta} = a - ib = r\left(\cos\theta - i\sin\theta\right)$ is called the complex conjugate number.