NAME: ....Wilhelm....................

POINTS: ....

Dr. Fritz Wilhelm, Diablo Valley College, Physics Department
e-mail: fredmaya@pacbell.net ; Phone: DVC Extension: 2403

chapters 12, 13, 14 in Giancoli.
May 3, 2016

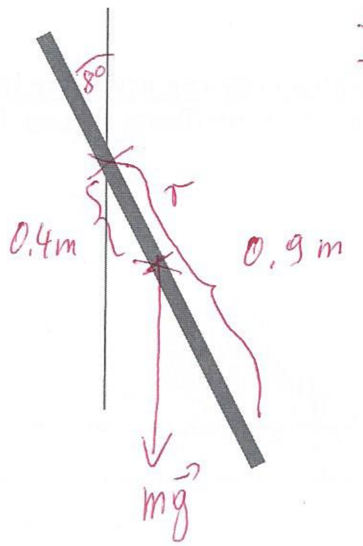
NOTE: <u>NOTE: Explain briefly what you are doing and which laws you are using in every step. Write legibly</u> USE DRAWINGS! Use the correct number of significant figures. Use vector symbols, arrows.

Contents

1. [20] PHYSICAL PENDULUM

A 1.00 meter long thin stick of mass 250 grams is suspended at the 90.0 cm mark in such a way that it can freely rotate around this point in a vertical plane. It is released from an initial angle of 8.00 degrees with the vertical. $I_{cm} = \frac{1}{12} ml^2$

a)[6] First, calculate the moment of inertia of the stick around its axis of rotation. Derive the differential equation by applying Newton's 2nd law (torques) to the object. Draw in the forces and label them correctly. What is the distance r_A? Show it in the graph.



$$I_A = \frac{1}{12} m l^2 + m \cdot (0.4l)^2$$
$$= \left(\frac{1}{12} + 0.16\right) m \cdot 1 = 0.0608\ kgm^2$$
$$\vec{\tau} = \vec{r} \times \vec{F}; \quad \tau = -r\, m g \sin\theta = I_A \alpha$$
$$\ddot{\theta} + \frac{mrg}{I_A}\theta = 0 \quad \text{for } \theta < 0.2$$

b)[4] Calculate its period of oscillation.

$$\omega_0^2 = \frac{4\pi^2}{T^2} = \frac{mrg}{I_A}; \quad T^2 = \frac{4\pi^2}{mrg} \cdot I_A = \frac{4\pi^2 \cdot 0.0608}{0.25 \cdot 0.4 \cdot 9.8}$$
$$= 2.45; \quad \underline{\underline{T = 1.565\ s}}$$
$$\omega_0 = \frac{2\pi}{T} = 4.01\ s^{-1}$$

c)[10] What are the speeds of the two endpoints of the stick when the meter stick has its greatest angular velocity? Use the equations of SHM, not energy conservation.

Upper: $\theta = \theta_0 \cos\omega_0 t \quad \dot{\theta}_{max} = \omega_0\theta_0 \quad v_{max} = \omega_0 \theta_0 l_{1,2}$

$\theta_0 = \cancel{0.122}$
$= 0.1396$

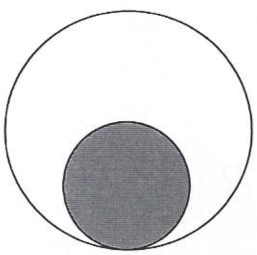
$$v_{max}^{up} = \underbrace{\omega_0 \theta_0}_{\cancel{0.490}} \cdot 0.1 = \cancel{0.04899\ m/s} = \cancel{4.90\ \frac{cm}{s}}$$
$$5.6\ cm/s$$

$$v_{down} = \cancel{44.1\ \frac{cm}{s}}\ 50.4\ cm/s$$

2. [10] CENTER OF MASS

[10] A uniform solid sphere of radius R has a smaller sphere of radius r=R/2 cut out of it. Find he

center of mass along the vertical axis (center of gravity) of the ~~disk~~ sphere with the cutout.



$$m_s = \frac{1}{8} M$$

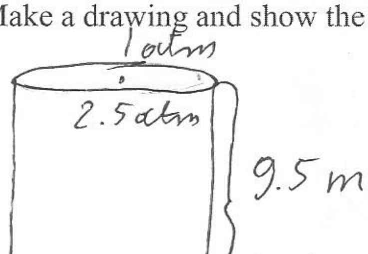
$$\cancel{M} \cdot R = \frac{R}{2} \cdot \frac{1}{8} \cancel{M} + y \cdot \frac{7}{8} \cancel{M}$$

$$\frac{7}{8} y = R - \frac{R}{16} = \frac{15}{16} R$$

$$y = \frac{8}{7} \cdot \frac{15}{16} R = \frac{15}{14} R$$

3. [10] WATERTANK

A cylindrical water tank of height 10.0 m is closed at the top and under a gauge pressure of 2.50 atm has a small hole on the outside 0.500 m above the bottom. Use Bernoulli's law to calculate the speed at which the water is being ejected from that hole.
Make a drawing and show the path of the water stream from top to bottom.



$$\cancel{\frac{1}{2} \rho v_1^2} + \rho g y_1 + P_1$$

$$\rho g y_1 + 2.5 \text{ atm}$$

$$= \frac{1}{2} \rho v_2^2 + \cancel{\rho g y_2} + \cancel{P_2}$$

$$\rho g y_1 + 2.5 \cdot 1.013 \cdot 10^5 = \frac{1}{2} \cdot 1000 \cdot v_2^2$$

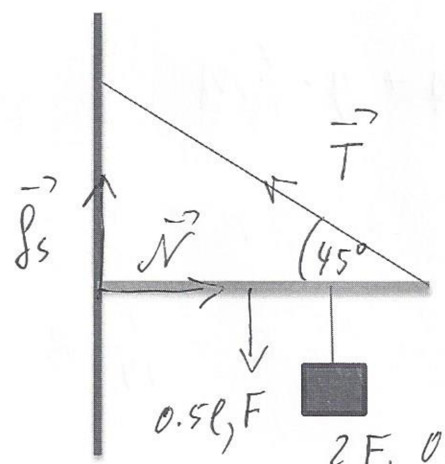
$$1000 \cdot 9.8 \cdot 9.5 + 2.5 \cdot 1.013 \cdot 10^5 = 500 \cdot v_2^2 \qquad 26.3$$

$$v_2^2 = \cancel{202.6}\ 692.7; \quad v_2 = \cancel{14.2}\ \frac{m}{s}$$

angle $\theta \neq 45^\circ$

4. [10] STATIC EQUILIBRIUM

[10] One end of a rod length L=1.00m and of weight F=100N is supported at its end by a cable attached to the vertical wall at an angle of ~~45.0~~ degrees between the horizontal rod and the cable. The other end of the rod rests against the wall, where it is held by friction. The coefficient of static friction is unknown. An additional weight of 2F hangs at point ¾ L, as measured from the wall along the rod. Find a relationship between the coefficient of static friction and the angle, which keeps the rod from slipping. The only variables in your answer are the angle and the coefficient of static friction. Clearly show all the exterior forces and torques you use.



$$T \sin\theta + f_s = 3F$$

$$N - T\cos\theta = 0$$

$$f_s = \mu_s T \cos\theta$$

$$T \sin\theta + \mu_s T \cos\theta = 3F$$

$$3F = \frac{3T}{2}\sin\theta$$

$$0.5\cancel{\ell}\cdot F + 0.75\cancel{\ell}\cdot 2F = T\sin\theta\cdot\cancel{\ell}$$

$$2F = T\sin\theta \;;\quad F = \frac{T}{2}\sin\theta$$

$$\cancel{T}\sin\theta + \mu_s \cancel{T}\cos\theta = \frac{3}{2}\cancel{T}\sin\theta$$

$$\mu_s \cos\theta = 0.5 \sin\theta$$

$$\tan\theta = 2\mu_s$$

$$\mu_s = 0.5\tan\theta$$

5. [10] ENERGY OF DAMPED OSCILLATIONS

Find the rate at which a damped spring oscillation with mass m, spring constant k, and damping factor b loses its energy.

First write down the differential equation for damped oscillation, then find the total energy of the spring, then take its derivative with respect to time.

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2, \quad \frac{dE}{dt} = kxv + mva$$
$$= v(kx + ma)$$
$$= v(-bv) = -bv^2$$

6. [10] COMPLEX NUMBERS

Find the three solutions of the complex equation for n=0, 1, 2

$z^3 = +3 + 4i$

In exponential form.

$$\theta = \text{Atan}\,\frac{4}{3} = 0.9273$$

$$z_{0,1,2} = 5^{\frac{1}{3}} e^{i(0.309 + n \cdot 2.094)}$$

$n = 0, 1, 2$

Calculate the solution for n=2 in algebraic form.

$$z_2 = 1.71\, e^{i\,4.4979} = 1.71 \cdot (\cos 4.4979 + i \sin 4.4979)$$
$$= -0.364 - 1.67$$

7. [10] ENERGY OF A SPRING

A 2.00 kg object connected to a spring with a force constant of 32.0 N/m oscillates on a horizontal, frictionless surface with an amplitude of 50.0 cm.
Find a) [3] the total energy of the system and b) [3] the speed of the object when the position x is 25.0 cm. Find c) [2] the kinetic energy and d) [2] the potential energy when the position is 10.0 cm.

$E = \frac{1}{2} k A^2 = 16 \cdot 0.5^2 = \underline{4.00\ J.}$ (a)

$4 = \frac{1}{2} k x^2 + \frac{1}{2} m v^2 = 16 \cdot 0.0625 = 1 + v^2$

$v^2 = 3; \quad v = \underline{1.732\ \frac{m}{s}}$ (b)

$K = E - \frac{1}{2} k x^2 = 4 - 16 \cdot 0.1^2 = \underline{3.84\ J.}$ (c)

$U = E - K = 4 - 3.84 = \underline{0.160\ J}$ (d)

8. [10] FORCED SHM OSCILLATIONS (NO FRICTION)

A spring with mass 2.00 kg and spring constant 32.0 N/m is forced to oscillate by an outside driver (cosine function) with F_0=0.500N and angular frequency 3.9 radians/s. Derive the equation for the amplitude of the driven oscillation and calculate its value. Start with the differential equation from Newton's second law.

$m\ddot{x} + kx = F_0 \cos \omega_g t$

$x = A \cos \omega_g t \qquad A = \frac{F_0/m}{\omega_0^2 - \omega_g^2} = \frac{0.25}{16 - 3.9^2} = 0.316 m$

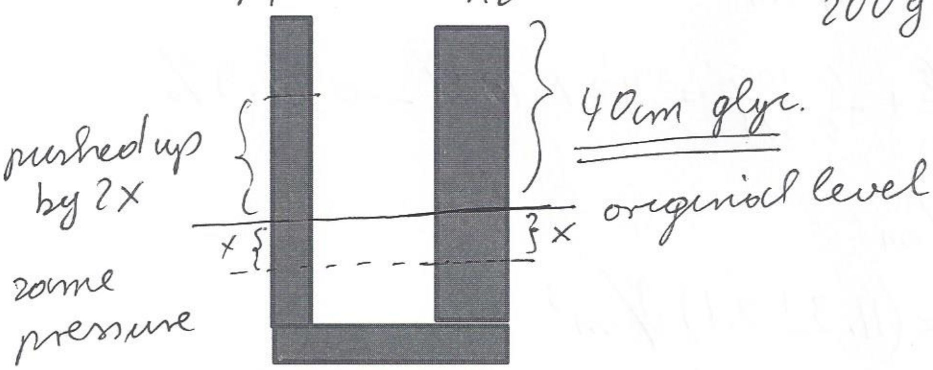
↑ from the original level.

9. [15] FLUID STATICS

[15] Mercury (ρ=13.6g/cm^3) is poured into a U-tube whose left arm has a cross-sectional area A_1 of 2.00 cm^2 and whose right arm has a cross-sectional area of A_2 of 4.00 cm^2. 200 grams of glycerin with density 1.25 g/cm^3 are then poured into the right arm pushing the mercury in this arm down and the mercury in the left arm up. a) Determine the height of the glycerin column in the right arm. b) By what distance does the mercury in the left arm rise? Explain which principles you are using. (4 significant figures.)

$A_1 = 2\,cm^2 \qquad A_2 = 4\,cm^2$

$$200\,g \text{ of glycerin} = \frac{200}{1.25}\,cm^3 = 160\,cm^3$$



$$3x \cdot 13.6 = 40 \cdot 1.25$$

$$x = \frac{40 \cdot 1.25}{3 \cdot 13.6} = 1.225\,cm$$

$2x = 2.45\,cm$ = distance rise of Hg.

10. [10] ARCHIMEDES PRINCIPLE

A plastic sphere is submersed 60% in water. The same substance is immersed by 40% in another unknown liquid. What is the density of the unknown liquid? Show detailed calculations with the principles applied.

$$\rho_L \cdot V_{im} = \rho \cdot V \;; \quad \rho = \frac{V_{im}}{V}\rho_L = 0.6\,g/cm^3$$

$$\rho_L' \cdot V_{im} = 0.6 \cdot V$$

$$\rho_L' = 0.6 \cdot \frac{V}{V_{im}} = \frac{0.6}{0.4} = \frac{6}{4} = \frac{3}{2} = 1.5\,g/cm^3$$

11. [10] UNCERTAINTY CALCULATIONS

You measure the dimensions of a circular cylinder with diameter (2.256+/-0.006) cm, height (5.621+/-0.005) cm, and mass (253.3+/-0.8)g.
Find the relative error, the absolute error, and the density of this cylinder with the correct number of significant figures.

$$V = \pi \frac{d^2 h}{4} = \frac{\pi \cdot 2.256^2 \cdot 5.621}{4} = 22.469\, cm^3$$

$$\rho = \frac{253.3}{22.469} = 11.2734\, g/cm^3$$

$$\frac{\Delta\rho}{\rho} = \frac{0.8}{253.3} + 2 \cdot \frac{0.006}{2.256} + \frac{0.005}{5.621} = 9.37 \cdot 10^{-3} \rightarrow 0.9\%$$

$$\Delta\rho = 0.106 \rightarrow 0.1\, g/cm^3$$

$$\rho = (11.3 \pm 0.1)\, g/cm^3$$

Formulas:

$$a) F_{damping} = -bv(t) \equiv -b\dot{x}$$
$$b) F = -kx - b\dot{x} = m\ddot{x} \text{ or} \qquad (1.1)$$
$$c) \ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0; \omega_0^2 = \frac{k}{m}; x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega_1 t + \varphi)$$

simple pendulum: $\omega_0^2 = \frac{g}{l}$ physical pendulum $\omega_0^2 = \frac{mgr_A}{I_A}$

(1.2) $\hat{z} = Ae^{i\theta} = A(\cos\theta + i\sin\theta) = a + ib$; with $A = \sqrt{a^2 + b^2} = \sqrt{\hat{z}\hat{z}^*}$ $\theta = \arctan\frac{b}{a}$

(1.3)
$$\hat{z}^k = re^{i(\theta + 2n\pi)} = a + ib = \cos(\theta + 2n\pi) + i\sin(\theta + 2n\pi) \text{ has k solutions:}$$
$$\hat{z}_k = r^{\frac{1}{k}} e^{i\frac{(\theta + 2n\pi)}{k}} \text{ for n=0,1...k-1}$$

$P = \frac{F}{A}$; the pressure in a liquid of depth y, measured from the surface is:

$$P = \rho g y; [P] = pascals = \frac{N}{m^2}; 1atm = 1.013 \times 10^5\, pa = 14.7 \frac{lbs}{in^2}$$

Pressure: $dF = PdA$

The buoyant force B equals the weight of the displaced liquid.

$$\frac{\rho}{\rho_{liquid}} = \frac{V_{liquid}}{V}; m = \rho V$$

Continuity equation;

$$A_1 v_1 = A_2 v_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Static equilibrium: $\sum_i \vec{F}_i = \vec{0}; \quad \sum_i \vec{\tau}_i = \vec{0}$