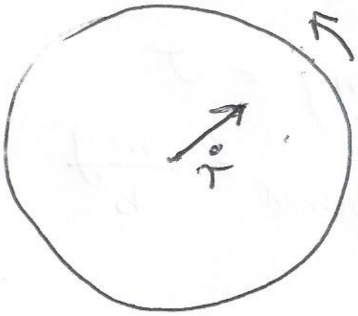
Rest of ch 5, Ch 6 Gravitation

- Coriolis derivation
- Terminal velocity
- Universal gravitation; gravitation inside the earth

Kepler's laws : - Ellipses

- $T^2 \propto r^3 \rightarrow \frac{T^2}{d^3} = \text{const.}$; comet
- $\frac{d\vec{\ell}}{dt} = \vec{0}$ Angular momentum



$\omega = \text{constant}$ $\dot{\omega} = 0$

$$\vec{r} = r \cdot \vec{U}_r \, ; \quad \frac{d\vec{r}}{dt} = \dot{r}\,\vec{U}_r + r \cdot \omega\,\vec{U}_\theta$$

$$\frac{d^2\vec{r}}{dt^2} = \cancel{\ddot{r}}\,\vec{U}_r + \dot{r}\,\omega\,\vec{U}_\theta + \dot{r}\,\omega\,\vec{U}_\theta + r\,\cancel{\dot{\omega}}\,\vec{U}_\theta - r\omega^2\,\vec{U}_r$$

$$+ 2\dot{r}\,\omega\,\vec{U}_\theta - r\omega^2\,\vec{U}_r$$

$$\vec{a}_{coriolis} = -2\dot{r}\,\omega\,\vec{U}_\theta \qquad \vec{F}_{cor} = m\,\vec{a}_{coriolis}$$

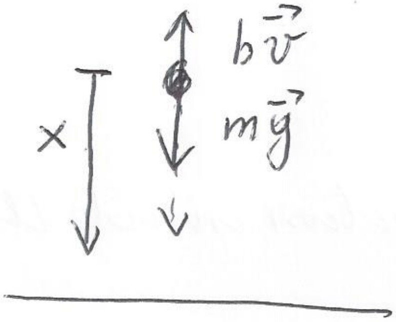
$$s = \frac{1}{2} a t^2$$

$$= -\cancel{2}\,\dot{r}\,\omega\, t^2$$

$2v\omega$

$\vec{F}_{friction} = b\vec{v}$ b is a constant

$b\vec{v} + m\vec{g} = m\vec{a}$ $[b] = \frac{kg}{s}$



x: $-bv + mg = m\frac{dv}{dt}$

"Simple" differential equation of the first order can be solved through separation of the variables.

$v(t=0) = 0$ initial condition

Terminal velocity when $a = 0$

$$-bv + mg = 0$$

$$v_{Terminal} = \frac{mg}{b}$$

$$\frac{\cancel{m}\,dv}{-b.v + mg} = \frac{dt}{m}$$

$$\frac{dv}{\cancel{b}(-v + \frac{mg}{b})} = \frac{b\,dt}{m};$$

$$\int \frac{dv}{v - \frac{mg}{b}} = \int -\frac{b}{m}\,dt$$

$$\ln\left(v - \frac{mg}{b}\right) = -\frac{b}{m}\cdot t + \text{const.}$$

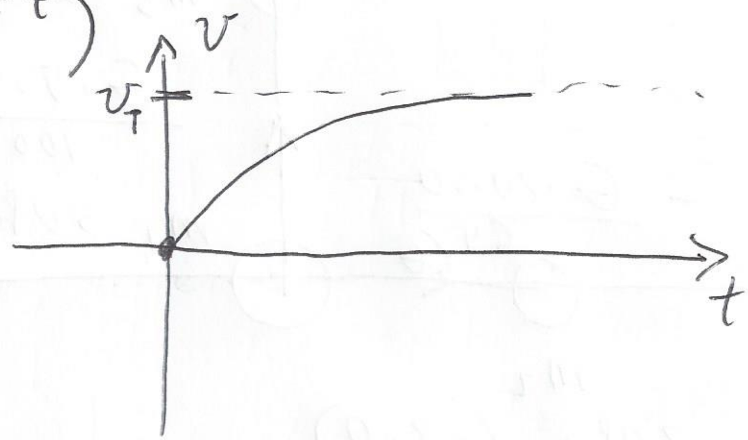
$$v - v_T = A e^{-\frac{b}{m}t}$$

$t=0$ $\quad 0 - v_T = A$

$$v - v_T = \cancel{-A e} = -v_T e^{-\frac{b}{m}t}$$

$$v - v_T = -v_T e^{-\frac{b}{m}t}$$

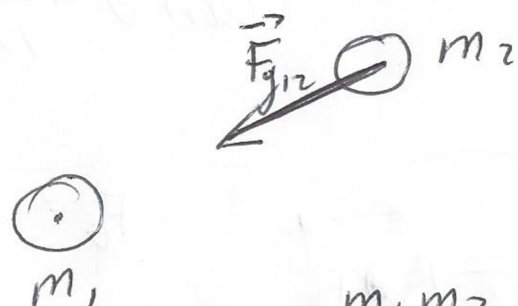
$$v = v_T \left(1 - e^{-\frac{b}{m}t}\right)$$



Universal Gravitation

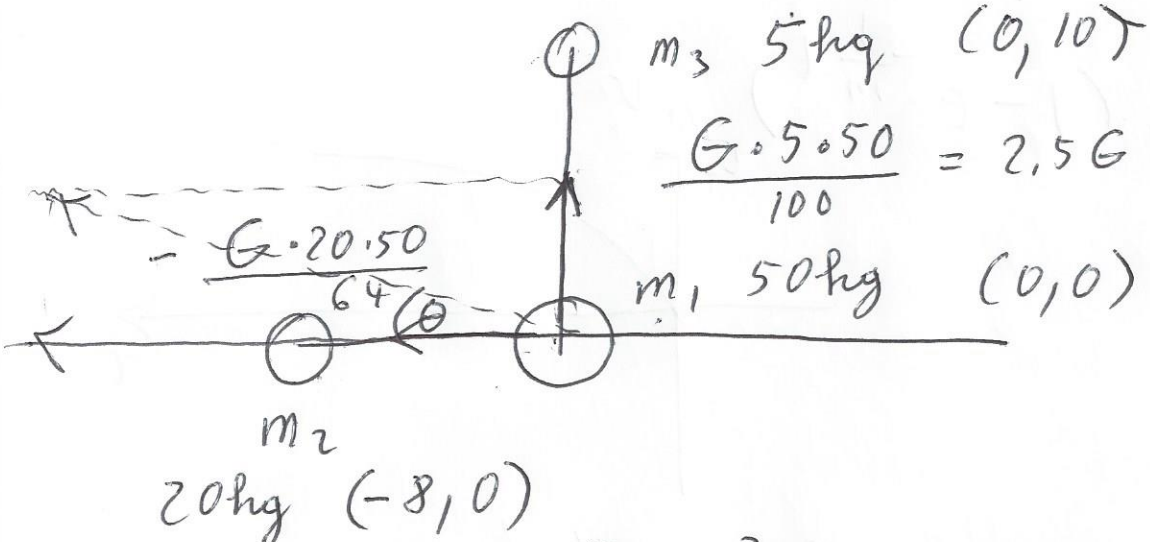
$$F_g = \frac{m_1 m_2 G}{r^2}$$

$$\vec{F}_g = -\frac{m_1 m_2 G}{r^2} \vec{U}_r$$



$$= \frac{-m_1 m_2 G}{x^2 + y^2} \frac{\langle x, y \rangle}{\sqrt{x^2+y^2}}$$

$$= -\frac{m_1 m_2 G}{(x^2 + y^2)^{3/2}} \langle x, y \rangle$$

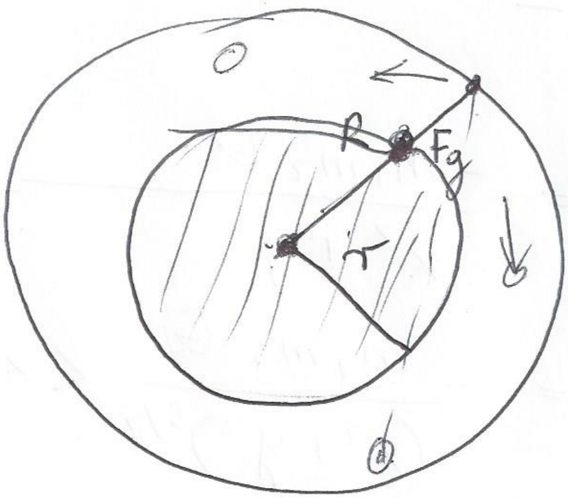


$$\vec{F} = \vec{F}_{13} + \vec{F}_{12} = \left\langle -\frac{1000}{64}, 2.5 \right\rangle G$$

$$= \langle -15.6, 2.5 \rangle G$$

$$|\vec{F}| = \sqrt{15.6^2 + 2.5^2} \cdot G$$

$$\tan\theta = \frac{2.5}{15.6} \; ; \; \underline{\underline{\theta = 9.1^\circ}}$$



$$F_\theta = \frac{G m_1 \cdot M'}{r^2}$$

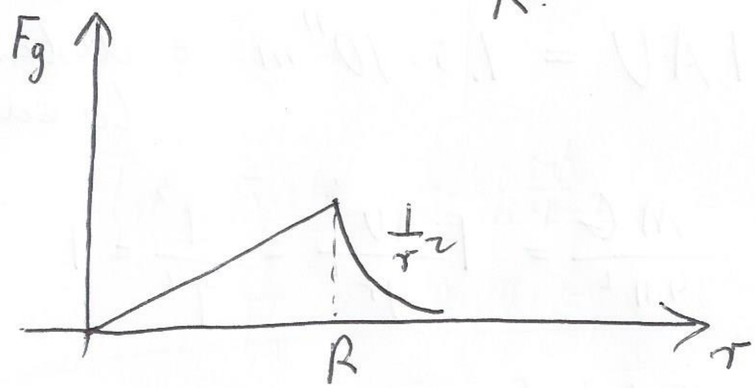
$$M' = \rho \cdot \frac{4\pi}{3} r^3$$

$$= \frac{G m_1 \cdot \rho \cdot \frac{4\pi}{3} r^3}{r^2}$$

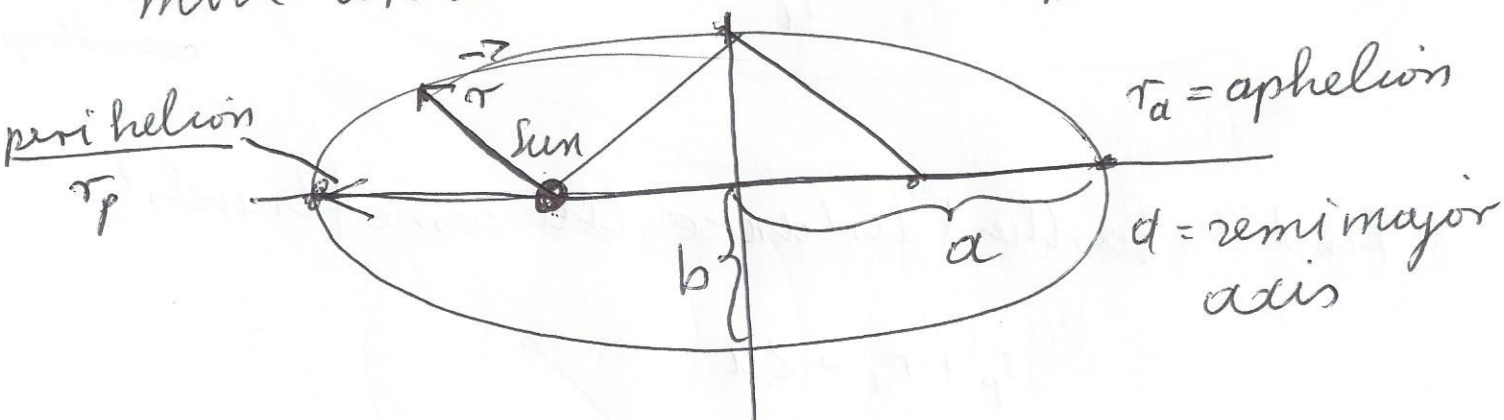
$$= G m_1 \rho \frac{4\pi}{3} \cdot r$$

$$\rho = \frac{M}{\frac{4\pi}{3} R^3} ; \quad F_g = G m_1 \frac{M}{\cancel{\frac{4\pi}{3}} R^3} \cdot \cancel{\frac{4\pi}{3}} \cdot r$$

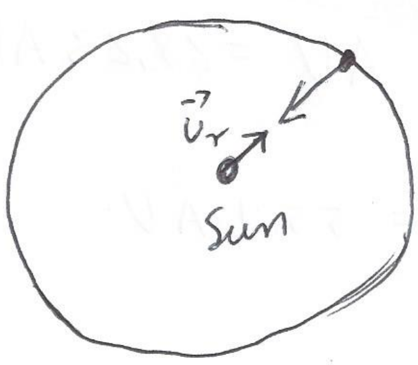
$$= \frac{G m_1 M}{R^3} \cdot r$$



<u>Kepler's first law</u>: All planets and comets move around the sun in ellipses.



<u>2nd law</u>: Assume a circular movement:



$$\vec{F}_g = m\vec{a}$$

$$+\frac{\cancel{m} \cdot M_S \cdot G}{r^2} = +\frac{\cancel{m} v^2}{r} = +\omega^2 r$$

$$= \frac{4\pi^2}{T^2} \cdot r$$

$$\frac{M_S \cdot G}{4\pi^2} = \frac{r^3}{T^2}$$

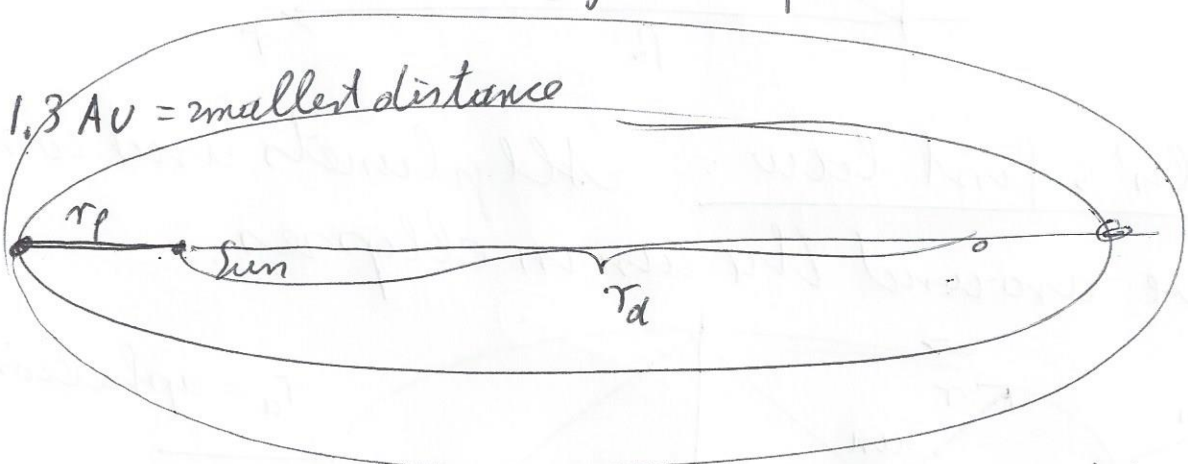
$$\boxed{\frac{MG}{4\pi^2} = \frac{a^3}{T^2}}$$

replacing r of a circle by a of an ellipse

$$\frac{1{,}99 \cdot 10^{30} \cdot 6{,}673 \cdot 10^{-11}}{4\pi^2} = \underline{\underline{3{,}36 \cdot 10^{18}}} \ \frac{m^3}{s^2}$$

$1\,AU = 1{,}5 \cdot 10^{11}\,m$ = distance from sun to earth

$$\frac{MG}{4\pi^2} = 1\,\frac{AU^3}{yr^2} = \frac{a^3}{T^2} = 1$$



150 yrs for a roundtrip.

Find the farthest distance the comet travels?

$$r_p + r_a = 2a$$

$$a^3 = T^2 = 150^2$$

$$a = 150^{2/3}\,AU = 28{,}23\,AU$$

$$r_a = 2a - r_p = 55{,}1\,AU$$