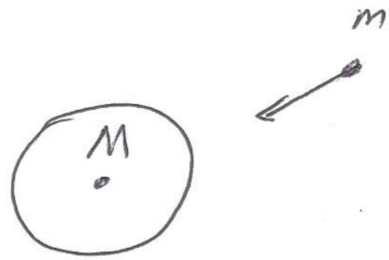
Ch 5 part 2 Lecture 7: Dynamics of Circular Motion

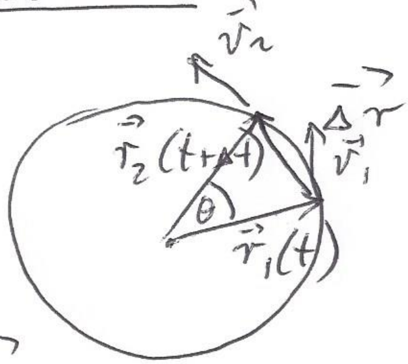
- What are the forces in circular motion?

- Vertical circle
- Conical pendulum
- Plane (pail of water)
- Embankment
- Amusement Park
- Turntable
- Coriolis

$$\vec{F} = - \frac{mMG}{r^2} \vec{U}_r$$

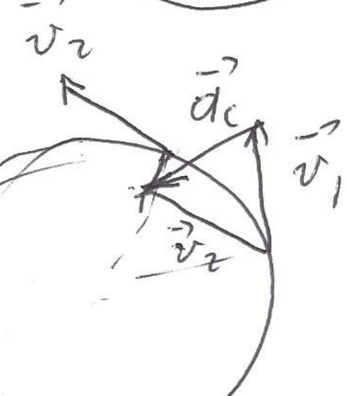


Circular Motion:



$$\lim \frac{\Delta \vec{r}}{\Delta t} = \vec{v}$$

$$\theta = \omega t$$



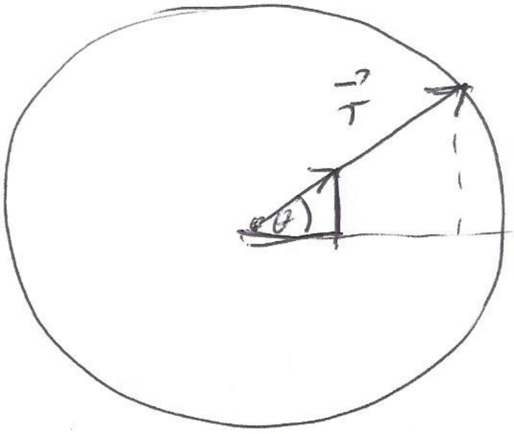
$\vec{v}$ tangential

$$v = \omega \cdot r = \frac{2\pi}{T} \cdot r$$

ω angular velocity

$$a_c = \omega^2 r = \frac{v^2}{r}$$

$\vec{a}_c$ points to the center



$$\vec{r} = r \cdot \vec{U}_r$$

$$\theta = \omega t$$

$$\vec{U}_r = \langle \cos\omega t, \sin\omega t \rangle$$

$$\frac{d\vec{U}_r}{dt} = \langle -\omega \sin\omega t, \omega \cos\omega t \rangle = \omega \underbrace{\langle -\sin\omega t, \cos\omega t \rangle}_{\vec{U}_\theta}$$

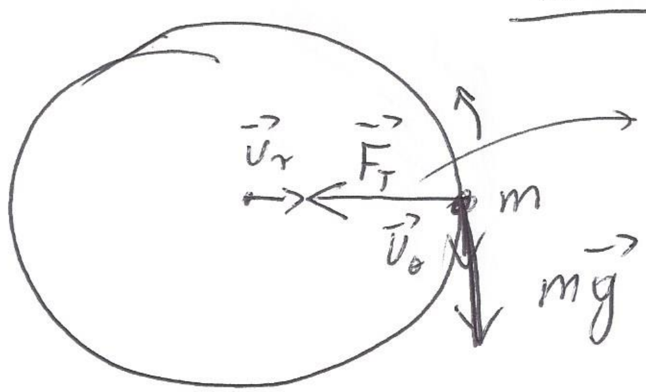
$$\frac{d\vec{U}_\theta}{dt} = \langle -\omega \cos\omega t, -\omega \sin\omega t \rangle = -\omega \langle \cos\omega t, \sin\omega t \rangle = -\omega \vec{U}_r$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \cancel{\dot{r}} \vec{U}_r + r \frac{d\vec{U}_r}{dt} = r \cdot \omega \vec{U}_\theta = \vec{v}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \cancel{\dot{r}} \omega \vec{U}_\theta + r \underbrace{\dot{\omega}}_{\alpha} \vec{U}_\theta + r\omega \underbrace{\frac{d\vec{U}_\theta}{dt}}_{-\omega \vec{U}_r}$$

$$\vec{a}_c = -\omega^2 r \vec{U}_r$$

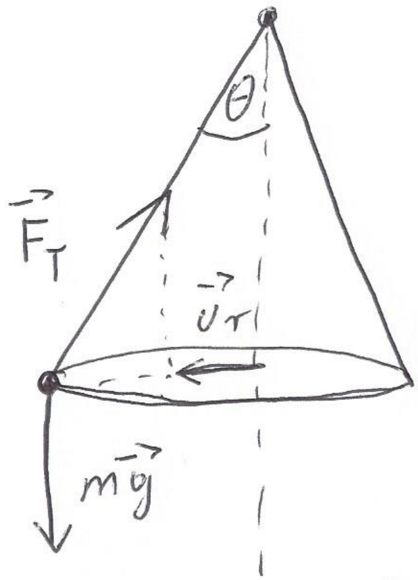
Forces



$$\vec{F}_T + m\vec{g} = m\vec{a}$$

$$\vec{U}_r : -F_T = -m\frac{v^2}{r}$$

$$\vec{U}_\theta : mg = m a_\theta = m \alpha \cdot r$$



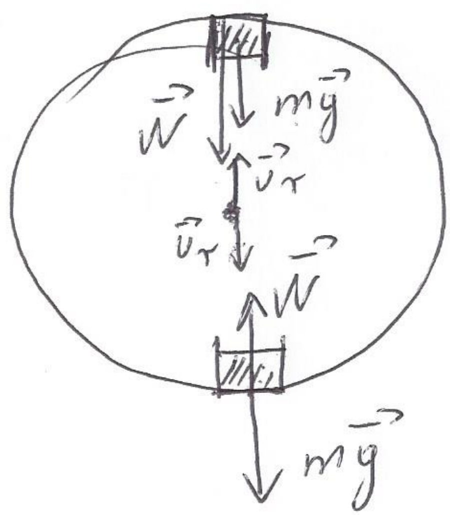
horizontal plane

$\vec{F}_T + m\vec{g} = m\vec{a}$

$\vec{u}_r : -F_T \sin\theta = -m\omega^2 r$

$\vec{u}_\theta$: no tangential acceleration

$z : F_T \cos\theta - mg = 0$



vertical circle, pail of water
m is the mass of water

top: $\vec{N} + m\vec{g} = m\vec{a}$

$\vec{u}_r : +N + mg = +m\frac{v^2}{r}$

$N_{top} = m\left(\frac{v^2}{r} - g\right)$

bottom: $\vec{N} + m\vec{g} = m\vec{a}$

$\vec{u}_r : -N + mg = -\frac{mv^2}{r}$

$N_{bot} = m\left(g + \frac{v^2}{r}\right)$

$r = 1m$

$\frac{v^2}{r} = g$

$v^2 = 9.8$

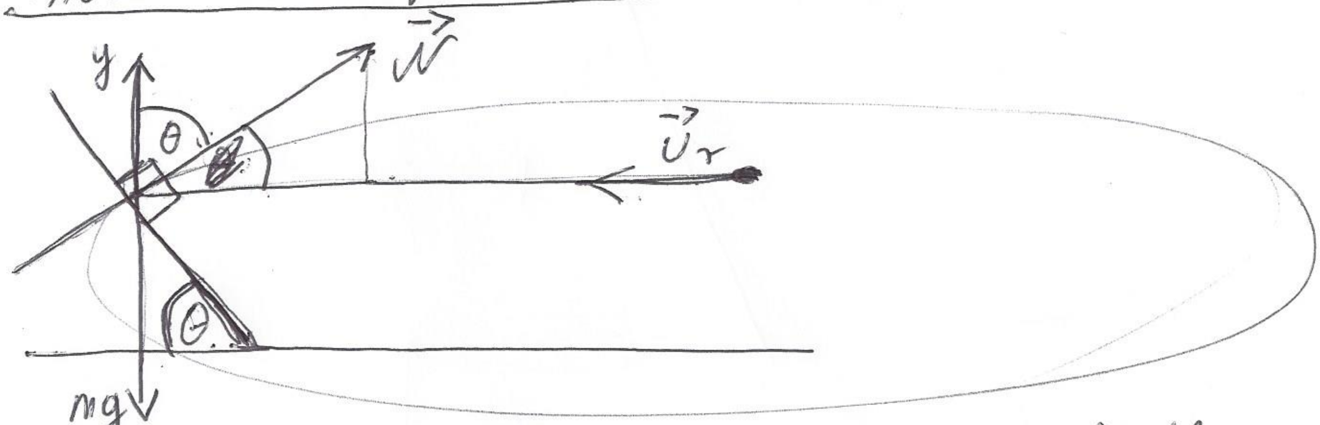
at $3.13 \frac{m}{s}$ the water drops out.

$\omega \cdot r = 3.13$ $\quad \omega = 3.13$ rps

$r = 500m$ $\quad \frac{v^2}{r} = 10g = 98$ $\quad v^2 = 98 \cdot 500$

$v = 221 \, m/s$ ✓

Embankment of a road curve.



At what angle do we have to embank the road so that ~~it a~~ the car does not skid off.

$$\vec{N} + m\vec{g} = m\vec{a}$$

$$\vec{u}_r : + N\sin\theta = + m\frac{v^2}{r}$$

$$y : N\cos\theta - mg = 0$$

$$N\sin\theta = m\frac{v^2}{r}$$

$$N\cos\theta = mg$$

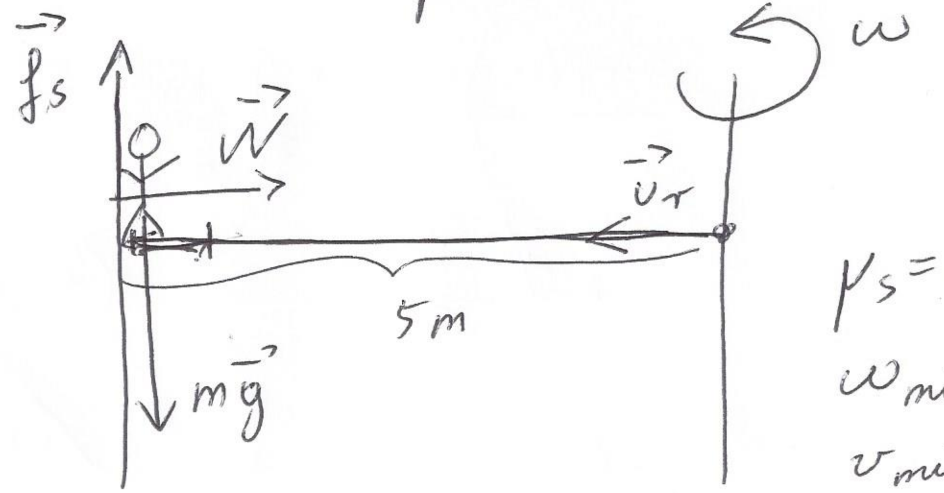
$$\tan\theta = \frac{v^2}{rg}$$

$r = 500m$

$v = 100\frac{m}{s}$

$$\tan\theta = ~~400^2~~ \frac{100^2}{500 \cdot 9{,}8} =$$

$$\underline{\underline{\theta = 64^\circ}}$$



$\mu_s = 0{,}5$

$\omega_{min} = ?$

$v_{min} = ?$

$$\vec{f}_s + m\vec{g} + \vec{N} = m\vec{a}$$

$$\vec{u}_r: \quad -N = -m\omega^2 r = -\frac{mv^2}{r} = -m\frac{4\pi^2}{T^2}\cdot r$$

$$y: \quad f_s - mg = 0$$

$$f_s = \mu_s \cdot N = mg$$

$$\mu_s \cdot m\omega^2 r = mg$$

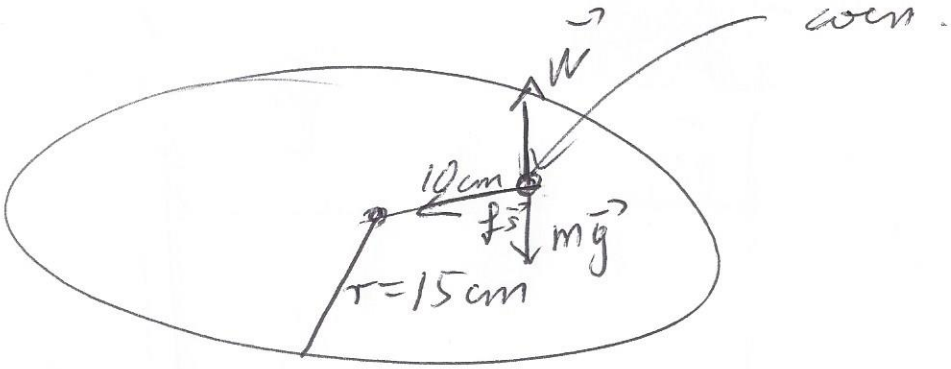
$$\omega^2 = \frac{g}{r\cdot\mu_s}$$

$$\omega = \sqrt{\frac{g}{r\cdot\mu_s}} = \sqrt{\frac{9{,}8}{5\cdot 0{,}5}} =$$

$$= 1{,}98 \text{ rad/s}$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = 3{,}17 \text{ s}$$



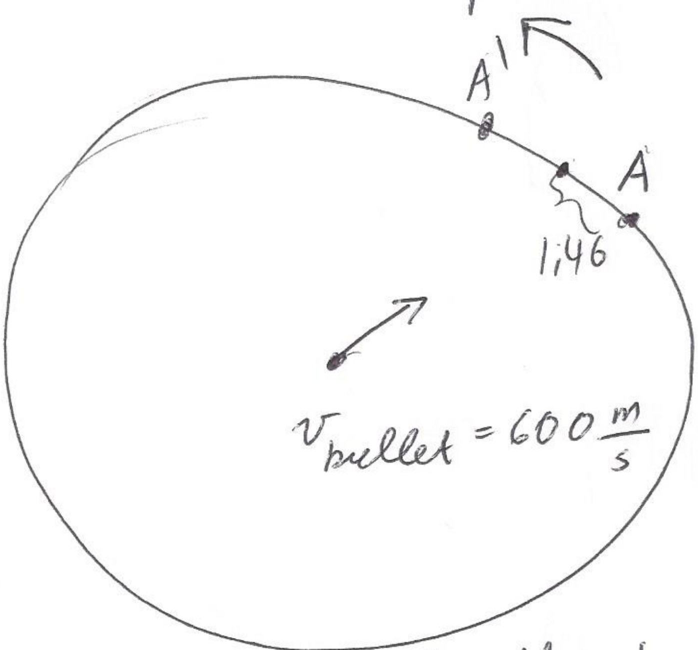
$$\vec{N} + m\vec{g} + \vec{f}_s = m\vec{a}$$

$$\vec{u}_r : \; -f_s = -m\frac{v^2}{r}$$

$$f_s = \mu_s \cdot N = \mu_s \cdot mg$$

$$\mu_s mg = \frac{m v^2}{r}$$

Coriolis acceleration and force.



$r = 1000 m$

$T = 2 \text{ hours} = 7200 s$

How long does it take the bullet to reach the target

$$t = \frac{x}{v} = \frac{r}{v_{bullet}} = \frac{1000 m}{600 m/s} = \frac{10}{6} s = 1.67 s$$

Δs = distance which point A moves on the circular path.

$$\Delta s = \omega \cdot r \cdot t = \frac{2\pi}{7200} \cdot 1000 \cdot 1.67 s$$

$$= 1.46 m$$