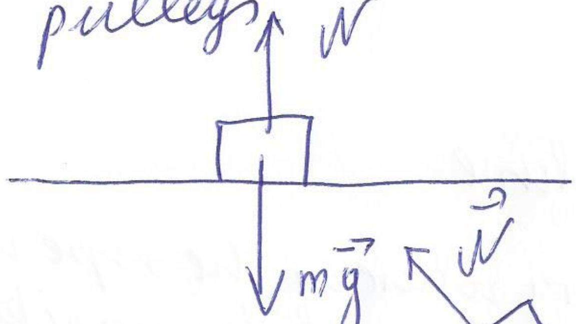
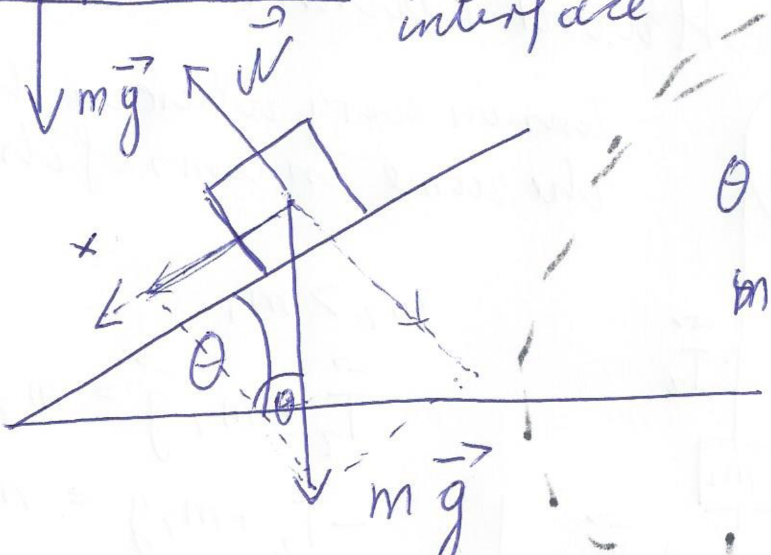
Chapter 4: Newton's laws

- Normal force
- Tension force in a rope (mass less)
- friction force
- pulleys

$g = 9.80 \frac{m}{s^2}$



$\vec{N}$ is perpendicular to the interface



$\theta = 36.9°$

$m = 5$ kg

$$\vec{N} + m\vec{g} = m\vec{a}$$

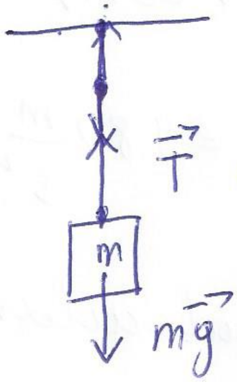
Choose the positive x-axis in the direction of the acceleration!

$x: \quad 0 + mg\sin\theta = ma$

$y: \quad N - mg\cos\theta = 0$

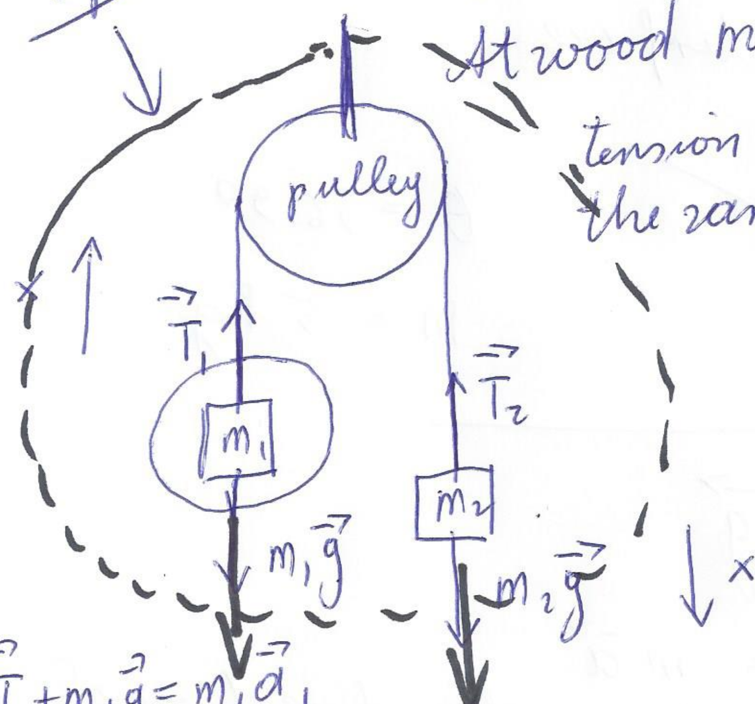
$$\underline{a = g\sin\theta}$$

tension in a rope



System

Atwood machine



tension every where in the rope is the same in tems of its magnitude

$m_2 > m_1$

$\vec{T}_2 + m_2\vec{g} = m_2\vec{a}_2$

$-T_2 + m_2 g = m_2 a_2$

$\vec{T}_1 + m_1\vec{g} = m_1\vec{a}_1$

$T_1 - m_1 g = m_1 a_1$

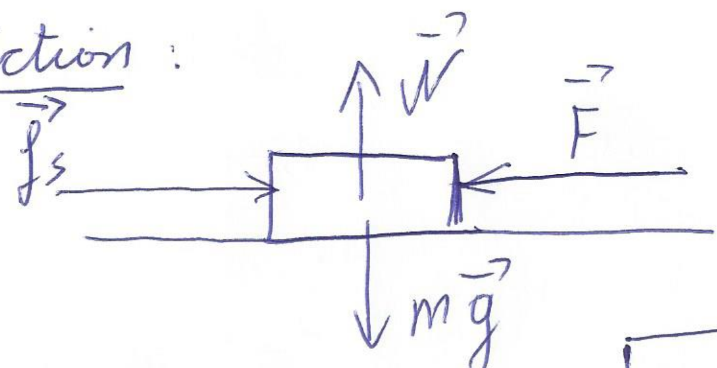
$$\left.\begin{aligned} a_1 &= a_2 = a \\ T_1 &= T_2 = T \end{aligned}\right\}$$

$$\begin{aligned} T - m_1 g &= m_1 a \\ -T + m_2 g &= m_2 a \\ \hline m_2 g - m_1 g &= (m_1 + m_2) a \end{aligned}$$

$a = \frac{m_2 - m_1}{m_2 + m_1} g$

$m_2 g - m_1 g = (m_1 + m_2) a$

friction:

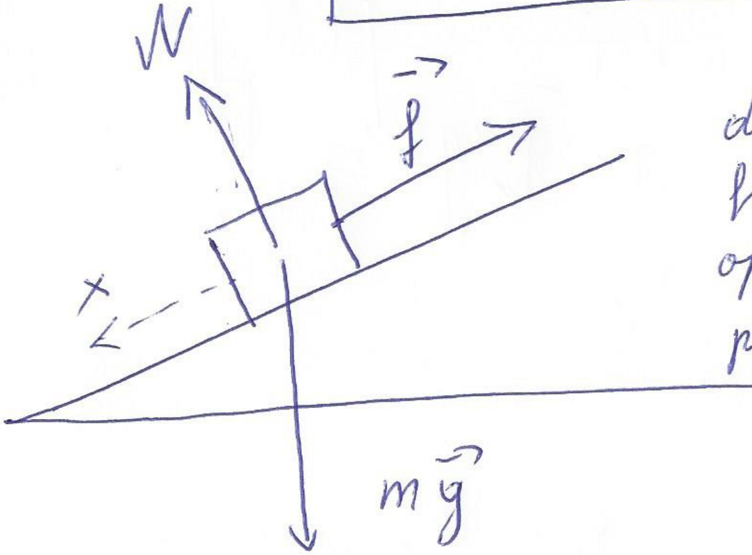


$\vec{f_s}$ = static friction $\Rightarrow$ $\boxed{f_s \leq \mu_s \cdot N}$

μ_s coefficient of static friction

in the moment the object starts to move we have kinetic friction.

$$\boxed{f_k = \mu_k \cdot N}$$



direction of the friction force is opposite to the potential or actual motion!

$\mu_K = 0.2$

$m = 10\,kg$

$\theta = 36.9°$

$\vec{N} + \vec{f_K} + m\vec{g} = m\vec{a}$

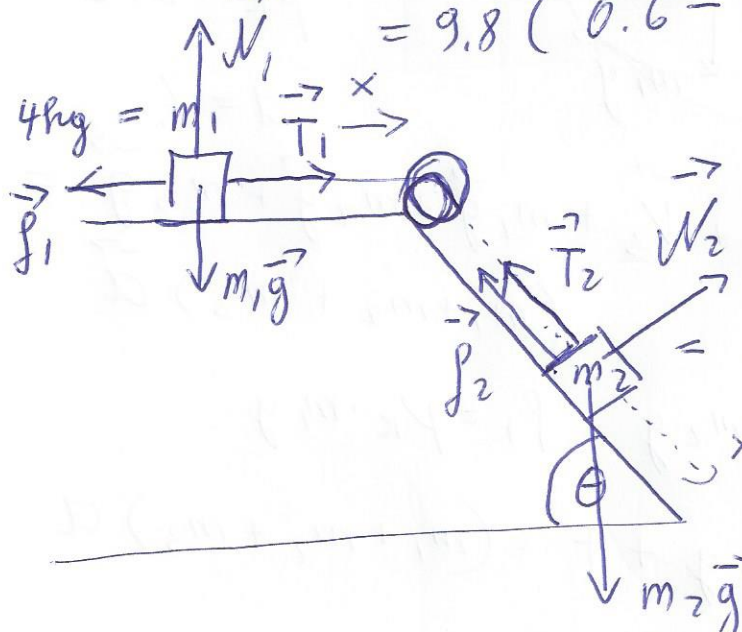
x: $-f + mg\sin\theta = ma$

y: $N - mg\cos\theta = 0$

$f = \mu_K \cdot N = \mu_K \cdot mg\cos\theta$

$-\mu_K mg\cos\theta + mg\sin\theta = ma$

$$a = g(\sin\theta - \mu_K \cos\theta)$$
$$= 9.8(0.6 - 0.2 \cdot 0.8)$$
$$= 9.8(0.6 - 0.16) = \underline{\underline{9.8 \cdot 0.44}}\ \frac{m}{s^2}$$



$\mu_K = 0.2$

$\theta = 36.9°$

$T_1 = T_2 = T$

$a_1 = a_2 = a$

$$\vec{f}_1 + \vec{N}_1 + \vec{T}_1 + m_1\vec{g} = m_1\vec{a}_1$$

x: $-f_1 + 0 + T_1 + 0 = m_1 a$

y: $N_1 - m_1 g = 0$

$f_1 = \mu_K \cdot N_1 = \mu_K \cdot m_1 g$

$-\mu_K m_1 g + T_1 = m_1 a$

$$\vec{T}_2 + \vec{f}_2 + \vec{N}_2 + m_2\vec{g} = m_2\vec{a}_2$$

x: $-T_2 - f_2 + 0 + m_2 g \sin\theta = m_2 a$

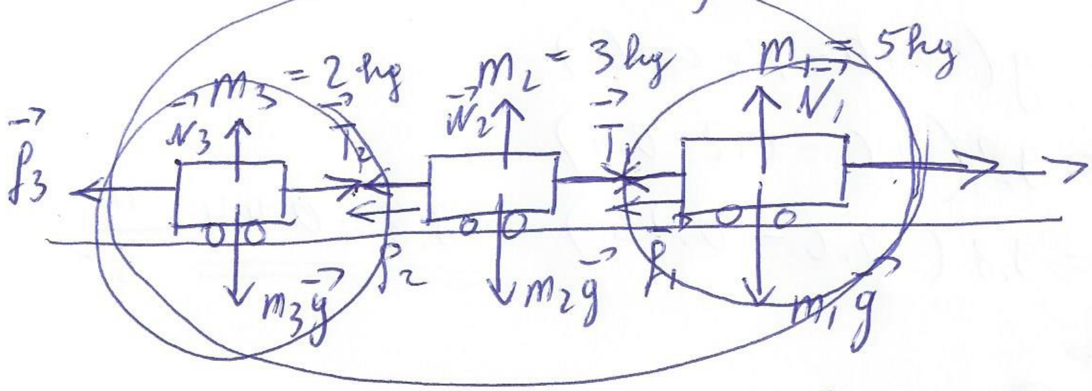
y: $N_2 - m_2 g \cos\theta = 0$

$f_2 = \mu_K \cdot m_2 g \cos\theta$

$-T_2 - \mu_K \cdot m_2 g \cos\theta + m_2 g \sin\theta = m_2 a$

$$-\mu_K m_1 g - \mu_K m_2 g \cos\theta + m_2 g \sin\theta = (m_1 + m_2)a$$

$$-0.2 \cdot 4 \cdot 9.8 - 0.2 \cdot 10 \cdot 9.8 \cdot 0.8 + 10 \cdot 9.8 \cdot 0.6 = 14\,a$$



$F = 200\,N$

$\mu_K = 0.2$

$a = ?$

$$\vec f_3 + \vec f_2 + \vec f_1 + \vec F + \vec N_1 + \vec N_2 + \vec N_3 + m_1\vec g + m_2\vec g + m_3\vec g = (m_1+m_2+m_3)\vec a$$

$f_3 = \mu_K m_3 g \quad f_2 = \mu_K m_2 g \quad f_1 = \mu_K \cdot m_1 g$

$x: \quad -(m_1+m_2+m_3)\mu_K g + F = (m_1+m_2+m_3)a$

$$200 - \underbrace{98\cdot 0.2}_{19.6} = 10\cdot a$$

$$\underline{a = 18\ m/s^2}$$

$$\vec T_1 + \vec f_1 + \vec N_1 + m_1\vec g + \vec F = m_1\vec a$$

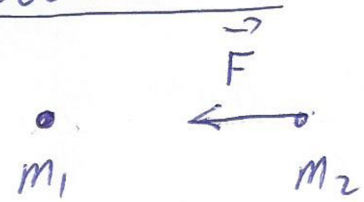
$-T_1 + F = m_1 a$
$-\mu_K \cdot m_1 g$

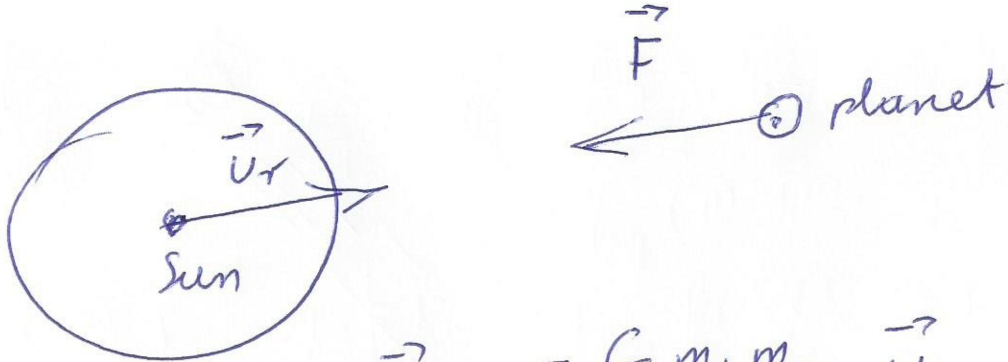
$T_1 = -m_1 a - \mu_K m_1 g + F$
$= -5\cdot 18 - 0.2\cdot 5\cdot 9.8 + 200$

$-f_3 + T_2 = m_2 a$

Newton's law of universal gravitation

$$F_g = \frac{G \cdot m_1 m_2}{r^2}$$





$$\vec{F} = \frac{-G m_1 m_2}{r^2} \vec{U}_r$$

$$r = \sqrt{x^2+y^2}$$

$$r^2 = x^2+y^2$$

$m_e = 5.98 \cdot 10^{24}$ kg

$$\vec{r} = \langle x, y \rangle \qquad \vec{U}_r = \frac{\langle x, y \rangle}{\sqrt{x^2+y^2}}$$

$m_s = 1.99 \cdot 10^{30}$ kg

$$\vec{F} = -\frac{G m_1 m_2}{x^2+y^2} \frac{\langle x, y \rangle}{\sqrt{x^2+y^2}}$$

$G = 6.673 \cdot 10^{-11}$

$$= \frac{-G m_1 m_2}{(x^2+y^2)^{3/2}} \langle x, y \rangle$$

$$= \frac{-G m_1 m_2}{r^2} \vec{U}_r$$