Lecture 21. Sound-waves

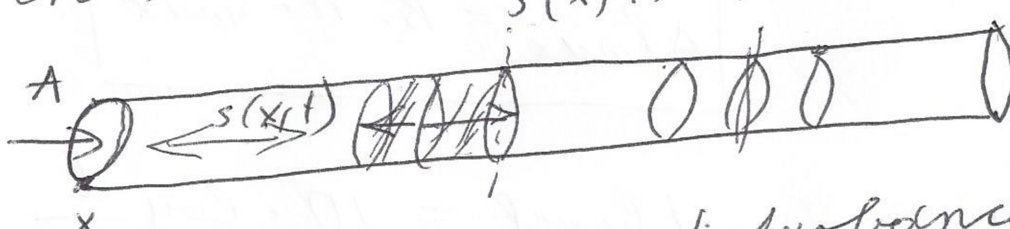
Soundwaves require a medium, a portion of which is being compressed above and beyond the regular pressure.

pressure for air : $1 atm = 1.013 \cdot 10^5 Pa$

$$1 Pa = \frac{1N}{m^2}$$

We call the additional pressure ΔP

Linear compression wave travelling through the air in a cylindrical column with lenght x and cross-section A. $S(x,t) \cdot A$ = changing volume



The additional pressure or disturbance travels within a volume $\Delta V_i = A \cdot \Delta x$

$y(x,t)$ for a transversal wave
$\downarrow$
$S(x,t)$ for a longetudinal wave.

$$S(x,t) = S_{max} \cancel{\sin} \cos(Kx - \omega t)$$

For a string we had $v^2 = \frac{F_T}{\mu}$

in water or steel $v = \sqrt{\frac{B}{\rho}}$ B = bulk modulus.

Relationship between $s(x,t)$ and $\Delta P(x,t)$

$$\Delta P = -B \cdot \frac{\Delta V}{V}; \quad \Delta V = A \cdot \Delta s(x,t)$$
$$V = A \cdot \Delta x$$

$$= -B \cdot \frac{\cancel{A} \cdot \partial s(x,t)}{\cancel{A} \cdot \partial x}$$

$$s = s_{max} \cdot \cos(kx - \omega t); \quad k = \frac{2\pi}{\lambda}$$

$$\frac{\partial s}{\partial x} = s_{max}(-k) \sin(kx - \omega t) \quad \omega = \frac{2\pi}{T} = 2\pi f$$

$$\Delta P_{max} = -B \cdot (-k)\, s_{max}$$

$$\boxed{\Delta P_{max} = B \cdot k \cdot s_{max}}$$

$$\beta = \text{sound level} = 10 \cdot \log \frac{I}{I_0}$$

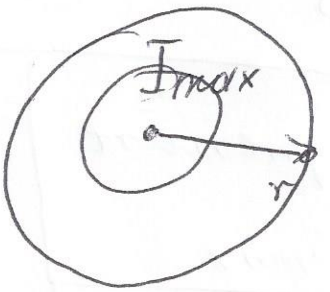
$$\underline{I} = \text{intensity} = \frac{\text{power}}{\text{cross-section}}$$

$$I_0 = 10^{-12} \frac{W}{m^2}$$ threshold of human hearing

$[\beta]$ = decibels

Power and intensity in the case of sound-waves

Energy $= E$ Power $= \frac{dE}{dt}$ intensity $= \frac{P}{A}$



$4\pi r^2$

$\frac{I_{max}}{4\pi r^2}$

Energy in a spring: $E = \frac{1}{2} k x_{max}^2$ (k → spring constant) $k = m\omega^2$

$$E = \frac{1}{2} m \omega^2 x_{max}^2$$

Energy on a vibrating string:

$$\Delta E = \frac{1}{2} \Delta m\, \omega^2 y_{max}^2$$

$$\Delta m = \mu \cdot \Delta x \qquad \mu = \text{linear density}$$

$$\Delta E = \frac{1}{2} \mu \cdot \Delta x\, \omega^2 y_{max}^2$$

$$\text{Power} = \frac{dE}{dt} = \frac{1}{2} \mu \underbrace{\frac{dx}{dt}}_{} \omega^2 y_{max}^2$$

$v =$ speed of propagation

Energy in a soundwave:

$$\Delta E = \frac{1}{2} \Delta m\, \omega^2 s_{max}^2 \; ; \qquad \Delta m = \rho \cdot \Delta V = \rho \cdot A \cdot \Delta x$$

$$= \frac{1}{2} \rho \cdot A \cdot \Delta x\, \omega^2 s_{max}^2$$

$$Power = \frac{dE}{dt} = \frac{1}{2}\rho \cdot v(\omega s_{max})^2 \cdot A$$

$$\boxed{Intensity = \frac{Power}{A} = \frac{1}{2}\rho v(\omega s_{max})^2 = I}$$

ΔP = excess pressure

$$\Delta P = B \cdot k \cdot s_{max}$$

$$\beta = 10 \log \frac{I}{I_0}$$

$\beta = 50$ decibels; sound wave in air frequency $f = 10^4\ Hz$

$$v^2 = \frac{B}{\rho}\ ;\ \rho = 1{,}25\ \frac{kg}{m^3}$$

$$v = 343\ \frac{m}{s}$$

speed of sound in air $= v = 331\ \frac{m}{s} \cdot \sqrt{1 + \frac{T_{celsius}}{273}}$

$I = ?$

$$50 = 10 \log \frac{I}{I_0}\ ;\ 5 = \log \frac{I}{I_0}$$

$$10^5 = \frac{I}{10^{-12}}\ ;\ I = 10^{-7}\ \frac{Pa}{m^2}$$

$S_{max} = ?$

$$I = \frac{1}{2} \rho v (\omega S_{max})^2$$

$$\frac{2I}{\rho v \omega^2} = S_{max}^2$$

$$\frac{2 \cdot 10^{-7}}{1{,}25 \cdot 343 (2\pi \cdot 10000)^2}$$

$$= \frac{2 \cdot 10^{-7} \cdot 10^{-8}}{1{,}25 \cdot 343 \cdot 4\pi^2} = 2 \cdot 10^{-15}$$

$$= 1{,}18 \cdot 10^{-19}$$

$$\underline{\underline{S_{max} = 3{,}4 \cdot 10^{-10}\ m}}$$

$$\Delta P = B \cdot k \cdot S_{max} \qquad \frac{\omega}{k} = v;\ k = \frac{\omega}{v}$$

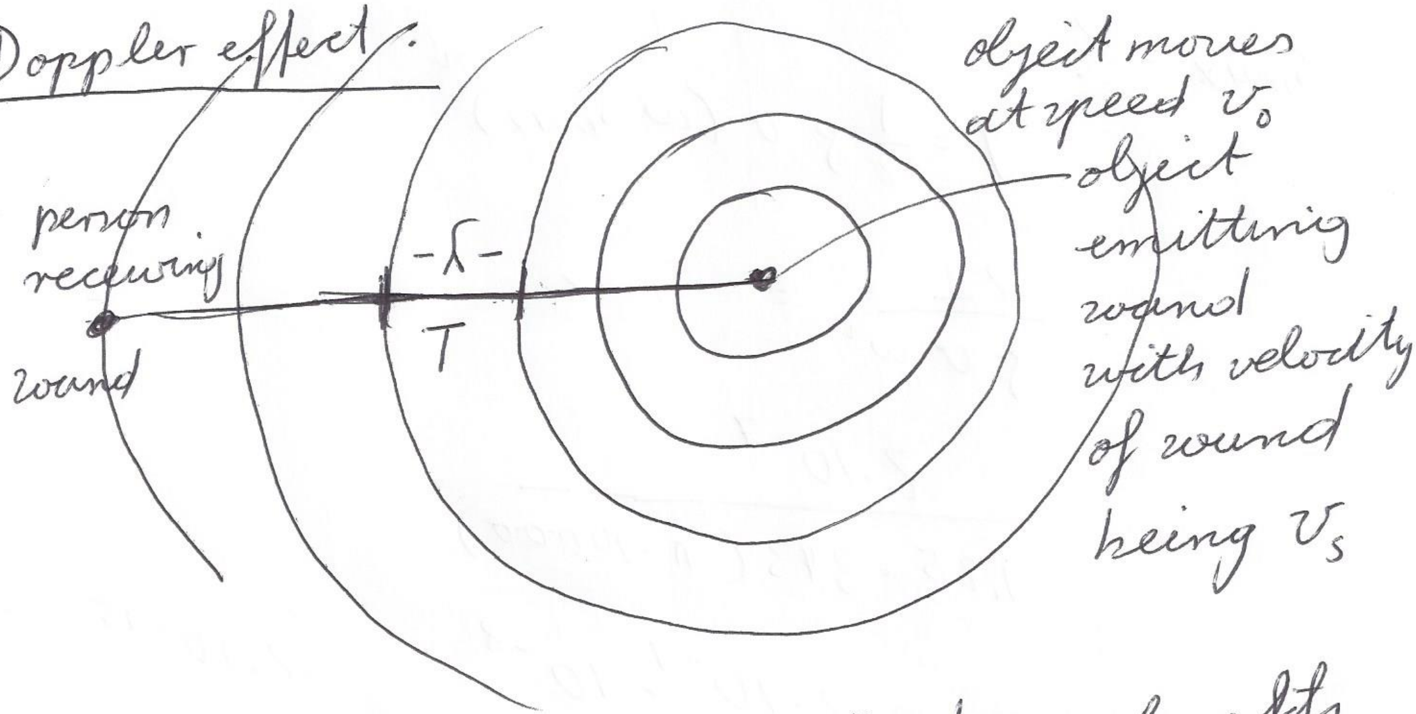
$$v^2 = \frac{B}{\rho}\ ;\ B = v^2 \cdot \rho$$

$$\Delta P = v^2 \cdot \rho \cdot \frac{\omega}{v} \cdot S_{max}$$

$$= 343^{\cancel{2}} \cdot 1{,}25 \cdot \frac{2\pi \cdot 10000}{\cancel{343}} \cdot 3{,}4 \cdot 10^{-10}$$

$$= 9{,}3 \cdot 10^{-3}\ Pa.$$

Doppler effect:



emitted wavelength is λ

$$v_s = \frac{\lambda}{T}$$

During the time one full wavelength is being received the source moves to the left by $\Delta x = v_0 \cdot T$

The received wavelength λ' is therefore shortened by Δx

$$\lambda' = \lambda - \Delta x = \lambda - v_0 T, \quad T = \frac{\lambda}{v_s}$$

$$= \lambda - v_0 \cdot \frac{\lambda}{v_s}$$

$$\lambda' = \lambda\left(1 - \frac{v_0}{v_s}\right)$$

$$= \lambda\left(\frac{v_s - v_0}{v_s}\right)$$

$$\frac{\lambda' \cdot f' = v_s}{\lambda \cdot f = v_s}$$

$$f' = \frac{v_s}{\lambda'} = \frac{v_s \quad v_s}{\lambda(v_s - v_0)} \qquad \frac{v_s}{\lambda} = f$$

approaching $f' = f\dfrac{v_s}{v_s - v_0}$; $\lambda' = \lambda\left(\dfrac{v_s - v_0}{v_s}\right)$

receding $\lambda' = \lambda\left(\dfrac{v_s + v_0}{v_s}\right)$

A siren with a frequency $= 15000$ Hz
approaches at 70 mph $= 70 \cdot \dfrac{1610}{3600}\,\dfrac{m}{s}$

$= 31.3\,\dfrac{m}{s}$

$$f' = 15000\left(\frac{343}{343 - 31.3}\right)$$

$$\underline{\underline{= 16500 \text{ Hz}}} \; ; \; \lambda'$$

$$\lambda' \cdot f' = 343\,\frac{m}{s}$$

$$\lambda' = \frac{343}{16\,500} = \underline{\underline{2.08 \text{ cm}}}$$

Relativistic doppler effect.

$$\lambda' = \lambda \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$$ blue shift

approaching

$$\lambda' = \lambda \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$ red shift

visible light λ between 350 and 700 nm.
10^{-9} m

We measure specific spectral lines emitted by a galaxy.

Example: a line of 350 nm is shifted to 700 nm.

$$\left(\frac{700}{350}\right)^2 = \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}$$