Energy/power — Lect 20 — DSE, Standing waves

- No interference problems, no superposition on final.

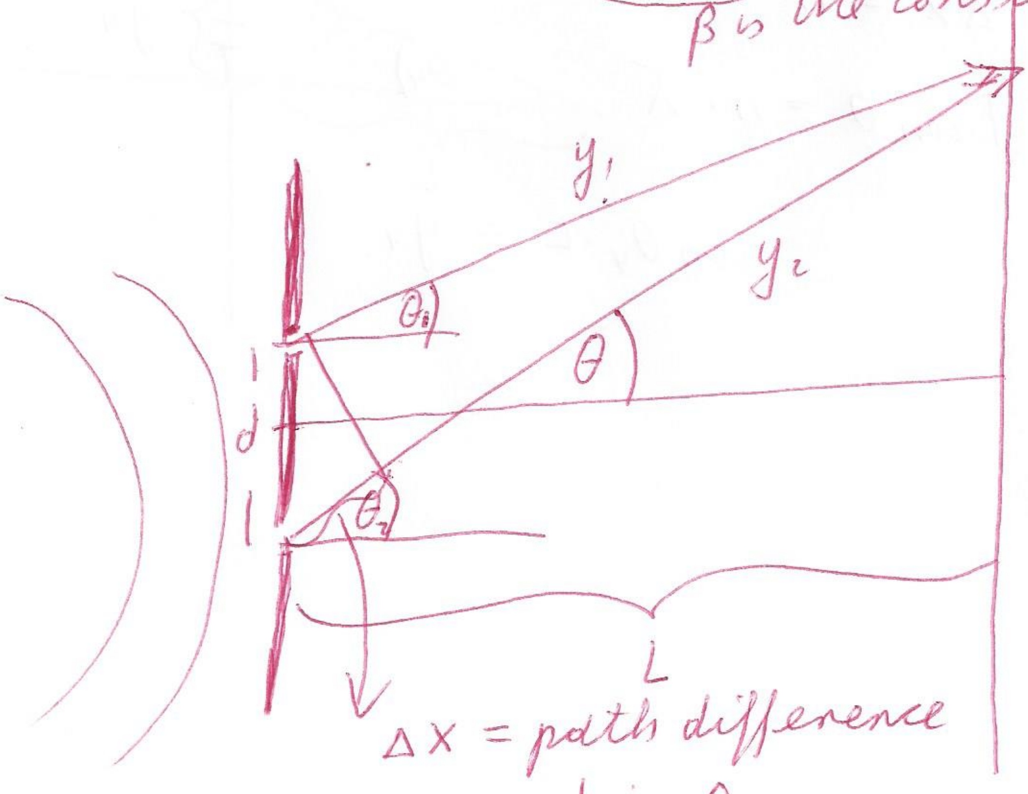
$$y = y_{max} \sin (kx - \omega t)$$
$$v^2 = \frac{F_T}{\mu}$$
is on final

$$y_1 = A \sin (kx - \omega t) = A \sin \theta_1$$
$$y_2 = A \sin (k_2 x - \omega_2 t) = A \sin \theta_2$$
$$y_1 + y_2 = 2A \cos \frac{\theta_2 - \theta_1}{2} \sin \left(\frac{\theta_2 + \theta_1}{2} \right)$$

DSE

$$y_1 + y_2 = 2A \cos \frac{\beta}{2} \sin \left(kx - \omega t + \frac{\beta}{2}\right)$$

β is the constant phaseshift



$\theta_1 = \theta_2 = \theta$ because $d \ll L$

$d \approx 0.02$ mm

$L = 2$ m

$\tan \theta \approx \sin \theta \approx \theta$

Δx = path difference $= d \sin \theta$

$2A\cos\frac{\beta}{2}$ is the new amplitude, therefore

if $\cos\frac{\beta}{2} = \pm 1$ we get a maximum in the intensity of the wave I = intensity = $\frac{\text{Power}}{\text{Area}}$

$$\frac{\beta}{2} = n\pi \qquad n = 1, 2, 3, \ldots$$

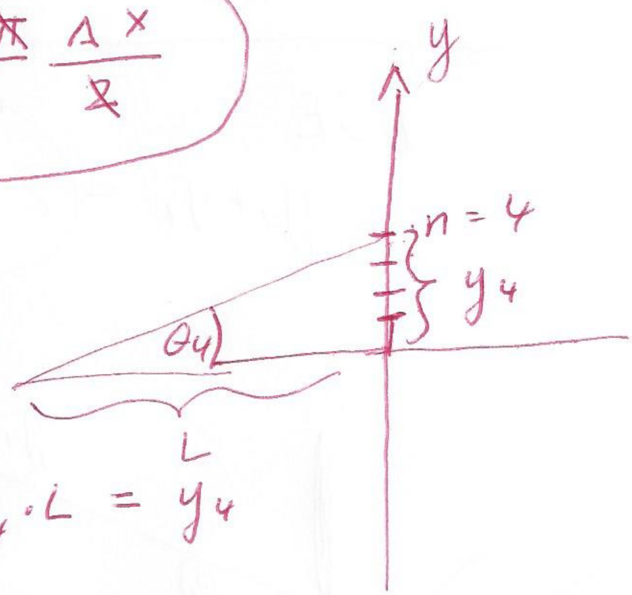
$$\beta = k\Delta x$$

$$y_2 = A\sin(K(x+\Delta x) - \omega t)$$

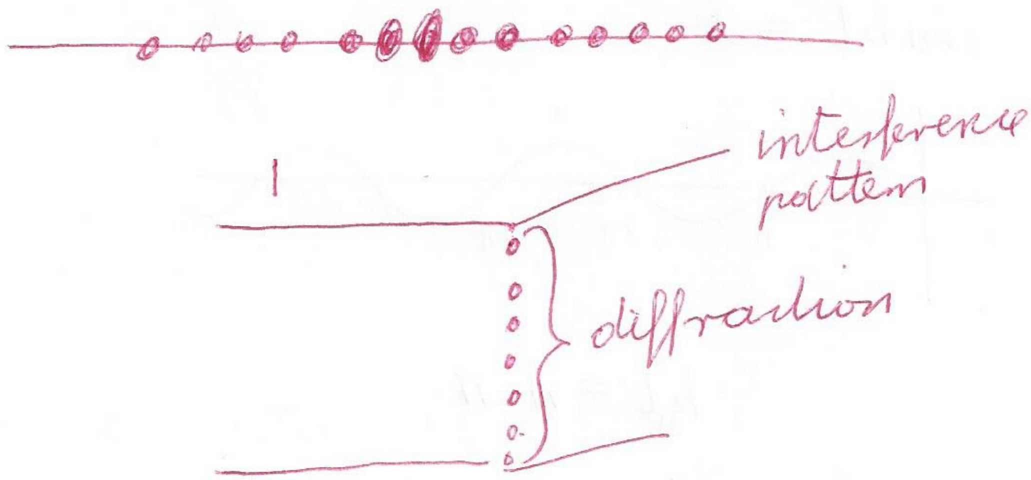
$$\frac{k\Delta x}{2} = n\cdot\cancel{\pi} = \frac{\cancel{2}\cancel{\pi}}{\lambda}\frac{\Delta x}{\cancel{2}}$$

$$\Delta x = n\cdot\lambda$$

$$d\sin\theta = n\cdot\lambda$$



$$\sin\theta_4\cdot L = y_4$$



Standing waves on a string:

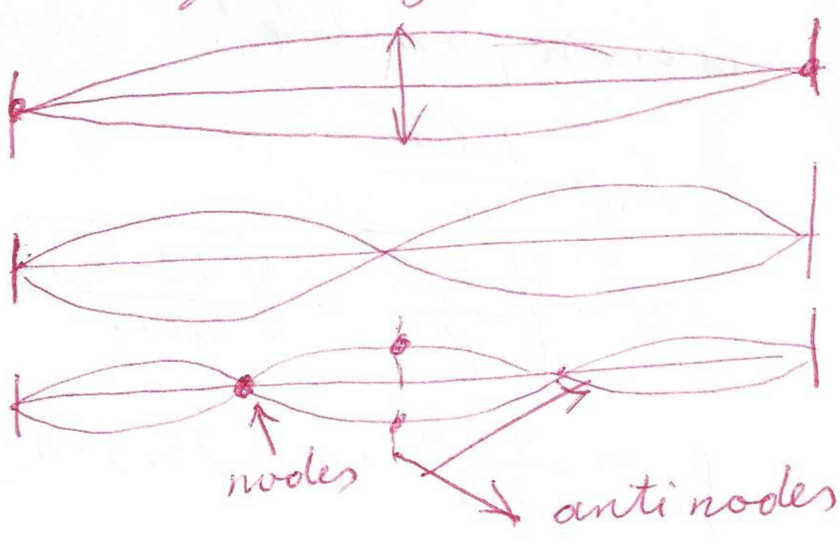
$\vec{mg}$ creates a tension F_T in the wire

$y_1 = A \sin(Kx + \omega t)$ right to left

$y_2 = A \sin(kx - \omega t)$ left to right

$y_1 + y_2 = 2A \cos \omega t \sin kx$

Standing wave.

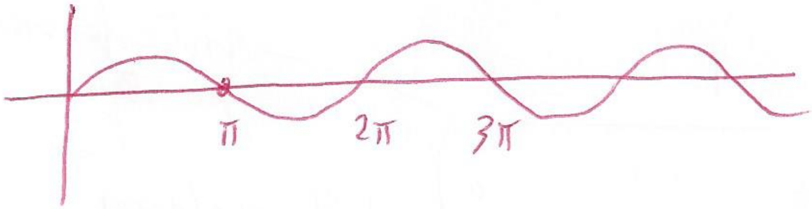


$x = L$

$\sin kL = 0$

<u>boundary condition</u>

$\sin kL = 0$



$$k_n L = n \cdot \pi$$

$$\frac{2\pi}{\lambda_n} \cdot L = n\pi$$

$$\lambda_n = \frac{2L}{n}$$ possible wavelengths leading to a standing wave.

$n = 1, 2, 3, 4 \ldots$

$$v = \lambda_n \cdot f_n$$

$$f_n = \frac{v}{\lambda_n} = \underbrace{\left(\frac{v}{2L}\right)}_{\text{fundamental frequency}} \cdot n$$

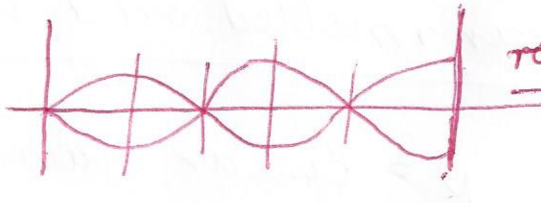
<u>Example:</u> Violin string $L = 0.90\,m$; $m = 13\,g$; $F_T = 30\,N$

What frequencies are possible? What wave lengths?

$$\lambda_n = \frac{1.8\,m}{n}$$

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{30 \cdot 0.9}{0.013}}\ \frac{m}{s} = 45,6\ \frac{m}{s}$$

$$f_n = \frac{45,6}{1.8} \cdot n = 25,3 \cdot n\ \underline{\text{Hertz}}$$



reflection from open space.

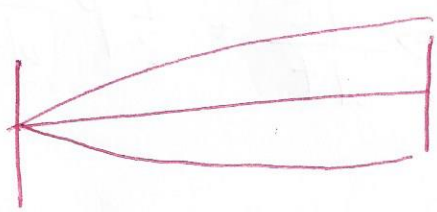
$$\cancel{\cos k}\ \sin kL = \pm 1$$

$$kL = \frac{2n+1}{2}\pi \quad \text{odd multiples of } \frac{\pi}{2}$$

$$k_{2n+1} \cdot L = \frac{2n+1}{2}\pi$$

$$\frac{2\cancel{\pi}}{\lambda_{2n+1}} \cdot L = \frac{2n+1}{2}\cancel{\pi}$$

$$\lambda_{2n+1} = \frac{4L}{2n+1} \qquad n = 0, 1, 2, \ldots$$



$$n = 0 = 4L$$

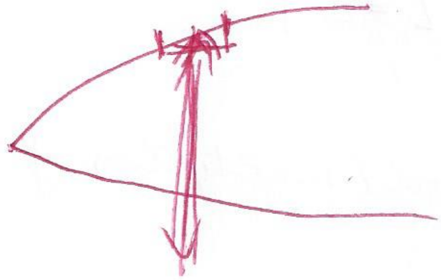
$$n = 1 \quad \lambda_3 = \frac{4L}{3}$$

$$n = 2 \quad \lambda_5 = \frac{4L}{5}$$

$$f_{2n+1} = \frac{v}{\lambda_{2n+1}} = \left(\frac{v}{4L}\right)\cdot(2n+1)$$

Energy and Power transmitted on a string:

$\Delta m = \mu \cdot \Delta x \qquad \mu =$ linear mass density



$$\Delta E_{max} = \frac{1}{2} k y_{max}^2 \qquad k = m\omega^2$$

$$= \frac{1}{2} \Delta m\, \omega^2 y_{max}^2 ; \qquad \Delta m = \mu \cdot \Delta x$$

$$\Delta E_{max} = \frac{1}{2} \mu \Delta x (\omega y_{max})^2$$

$$\text{Power} = \frac{\Delta E}{\Delta t} \rightarrow \frac{dE}{dt} = \frac{1}{2} \mu \frac{dx}{dt} (\omega y_{max})^2$$

Power transmitted on a string.

$$P = \frac{1}{2} \mu v (\omega y_{max})^2$$

$f_{10} = 253$ Hertz $\quad y_{max} = 0.8$ cm

$$P = \underbrace{\frac{1}{2} \frac{0.013}{0.9} \cdot 45.6 \tfrac{m}{s}}_{0.329} (2\pi \cdot 253 \cdot 0.008)^2 = \underline{\underline{53.26 \text{ W}}}$$