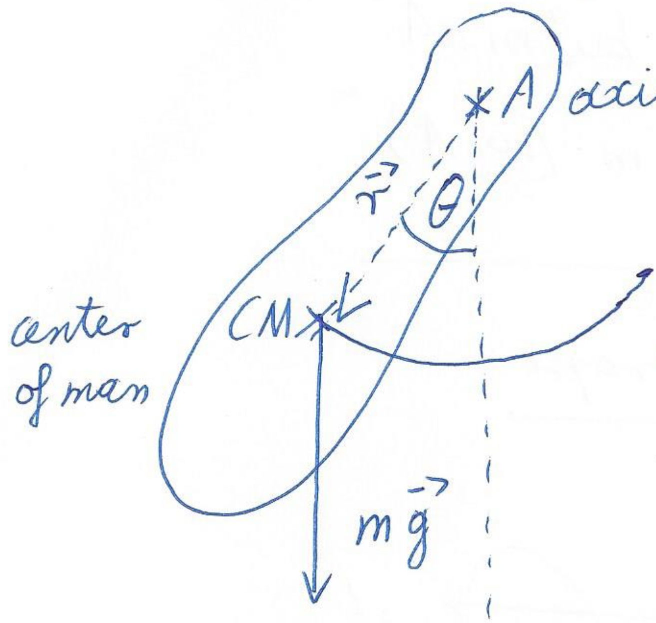
- Lect. 19 -

Oscillations: Physical Pendulum.



axis of rotation

$$\vec{\tau}_A = \vec{r}_A \times m\vec{g}$$

circular arc of rotation

$$\tau_A = -r_A \cdot mg \sin\theta$$

$$\sum \vec{\tau}_{ext} = I_A \vec{\alpha}$$

$$I_A \cdot \alpha = -r_A mg \sin\theta$$

$$I_A \ddot{\theta} + r_A mg \sin\theta = 0$$

$\sin\theta \approx \theta$ for $\theta < 12^\circ$

$$\ddot{\theta} + \frac{r_A mg}{I_A} \cdot \theta = 0$$

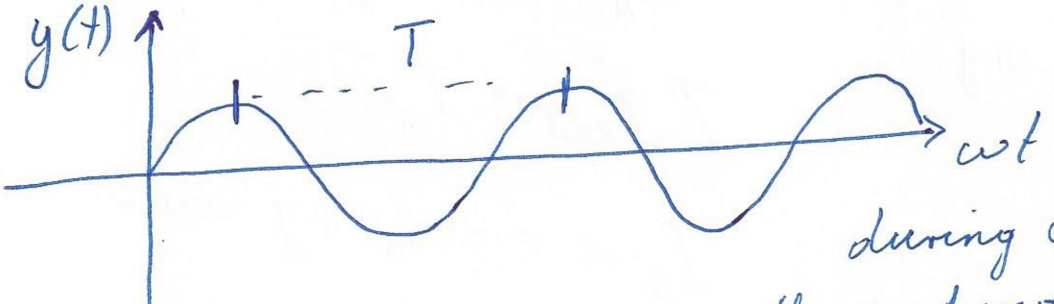
$$\omega_0^2 = \frac{r_A mg}{I_A} = \frac{4\pi^2}{T^2} = 4\pi^2 f^2$$

$$\theta(t) = \theta_0 \cos\omega_0 t$$

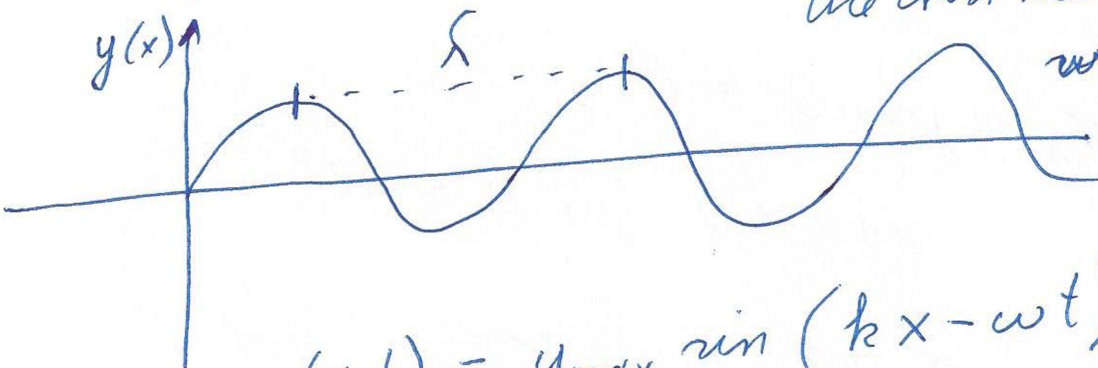
Power, energy of a spring:

$$E_{max} = \frac{1}{2} k A^2 = \frac{1}{2} k\omega_0^2 m \cdot A^2$$
$$= \frac{1}{2} m (\omega_0 A)^2$$

Waves on a string / rope:



during a period T the crest moves by a wavelength λ.



$+\phi$

$$y(x,t) = y_{max} \sin(kx - \omega t)$$

$\omega = \frac{2\pi}{T}$ $\quad k = \frac{2\pi}{\lambda}$ wave number

$$\frac{\partial^2 y(x,t)}{\partial t^2} = \alpha \frac{\partial^2 y(x,t)}{\partial x^2}$$

$$\frac{\partial y(x,t)}{\partial t} = \frac{\partial(y_{max} \sin(kx-\omega t))}{\partial t} = y_{max} \cos(kx-\omega t)(-\omega)$$

$$y(x,t) = y_{max} \sin(Kx-\omega t)$$

$$z = Kx - \omega t$$

$$\frac{\partial y}{\partial t} = \frac{dy}{dz} \cdot \frac{\partial z}{\partial t} = -\omega\, y_{max} \cos(Kx-\omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 y_{max} \sin(Kx-\omega t)$$

$$\frac{\partial y}{\partial x} = k\, y_{max} \cos(Kx-\omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 y_{max} \sin(Kx-\omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = a \frac{\partial^2 y}{\partial x^2}$$

$$-\omega^2 y_{max} \sin(Kx-\omega t) = a \cdot (-k^2) y_{max} \sin(\;)$$

$$\frac{\omega^2}{k^2} = a \quad \frac{4\pi^2 \lambda^2}{T^2 4\pi^2} = \frac{\lambda^2}{T^2} = v^2$$

$$\boxed{v = \frac{\lambda}{T} = \frac{\omega}{k} = f \cdot \lambda}$$

speed of propagation of the wave.

transversal wave.

$$y(x,t) = y_{max} \sin(Kx - \omega t)$$

wave travels in the direction of increasing x.
from left to right

$$y(x,t) = y_{max} \sin(kx + \omega t)$$

from right to left.

$$\sin(kx - \omega t) = \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

$$v = \cancel{k}\,\frac{\omega}{k}\ ; \quad k = \frac{\omega}{v}$$

$\sin(x - vt)$ satisfies the wave equation.

Superposition of waves.

Interference of waves.

$$A \sin(k_1 x_1 - \omega_1 t_1) + B \sin(k_2 x_2 - \omega_2 t_2)$$

$$t_1 = t_2 = 2\,sec$$

$$x_1 = x_2 \quad 1.5\,m$$

$$k_1 = \frac{3}{m} \quad k_2 = \frac{3.5}{m} \quad \omega_1 = \frac{1000}{s} \quad \omega_2 = \frac{2000}{s}$$

Add sine functions (waves) with the same amplitude:

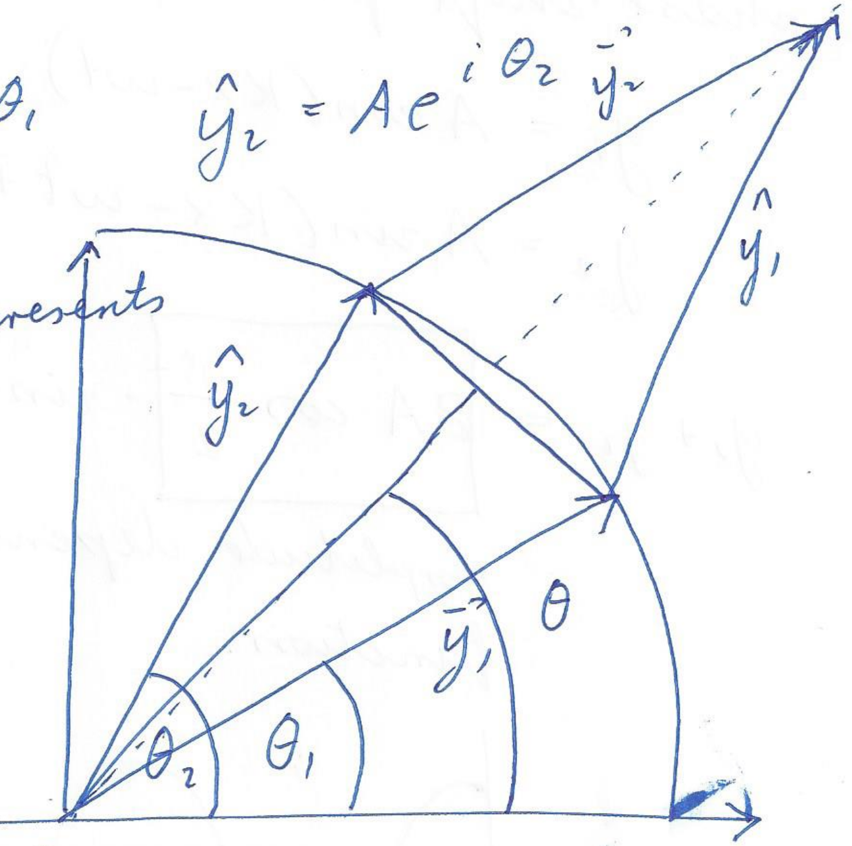
$$y_1 = A \sin(k_1 x_1 - \omega_1 t_1) = A \sin\theta_1$$

$$y_2 = A \sin(k_2 x_2 - \omega_2 t_2) = A \sin\theta_2$$

$$y_1 + y_2 = A\sin\theta_1 + A\sin\theta_2$$

$$= 2A\cos\left(\frac{\theta_2-\theta_1}{2}\right)\cdot\sin\left(\frac{\theta_2+\theta_1}{2}\right)$$

$$\hat{y}_1 = A e^{i\theta_1} \qquad \hat{y}_2 = A e^{i\theta_2}$$



The dotted vector represents the sum $\hat{y}_1 + \hat{y}_2 =$

$$\hat{y}_{res} = r e^{i\theta}$$

$$\frac{r}{2} = A\cos\frac{\theta_2-\theta_1}{2}$$

$$\theta = \theta_1 + \frac{\theta_2-\theta_1}{2} = \frac{\theta_2+\theta_1}{2}$$

$$\hat{y}_{res} = 2A \cos\frac{\theta_2-\theta_1}{2}\, e^{i\frac{(\theta_2+\theta_1)}{2}}$$

$$= 2A\cos\left(\frac{\theta_2-\theta_1}{2}\right)\left(\cos\frac{\theta_2+\theta_1}{2} + i\sin\frac{\theta_2+\theta_1}{2}\right)$$

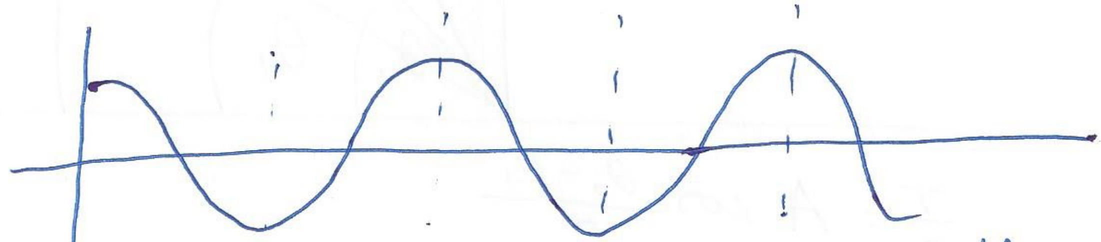
$$\boxed{y_1 + y_2 = 2A\cos\frac{\theta_2-\theta_1}{2}\sin\frac{\theta_2+\theta_1}{2}}$$

- Two waves are different by a constant phase shift β.

$$y_1 = A\sin(Kx-\omega t)$$
$$y_2 = A\sin(Kx-\omega t+\beta)$$

$$y_1 + y_2 = \boxed{2A\cos\frac{\beta}{2}}\cdot\sin\left(kx-\omega t+\frac{\beta}{2}\right)$$

amplitude depends on a cosine function.



the two waves extinguish each other whenever $\cos\frac{\beta}{2} = 0$; $\frac{\beta}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

destructive interference $= \frac{(2n+1)}{2}\pi$

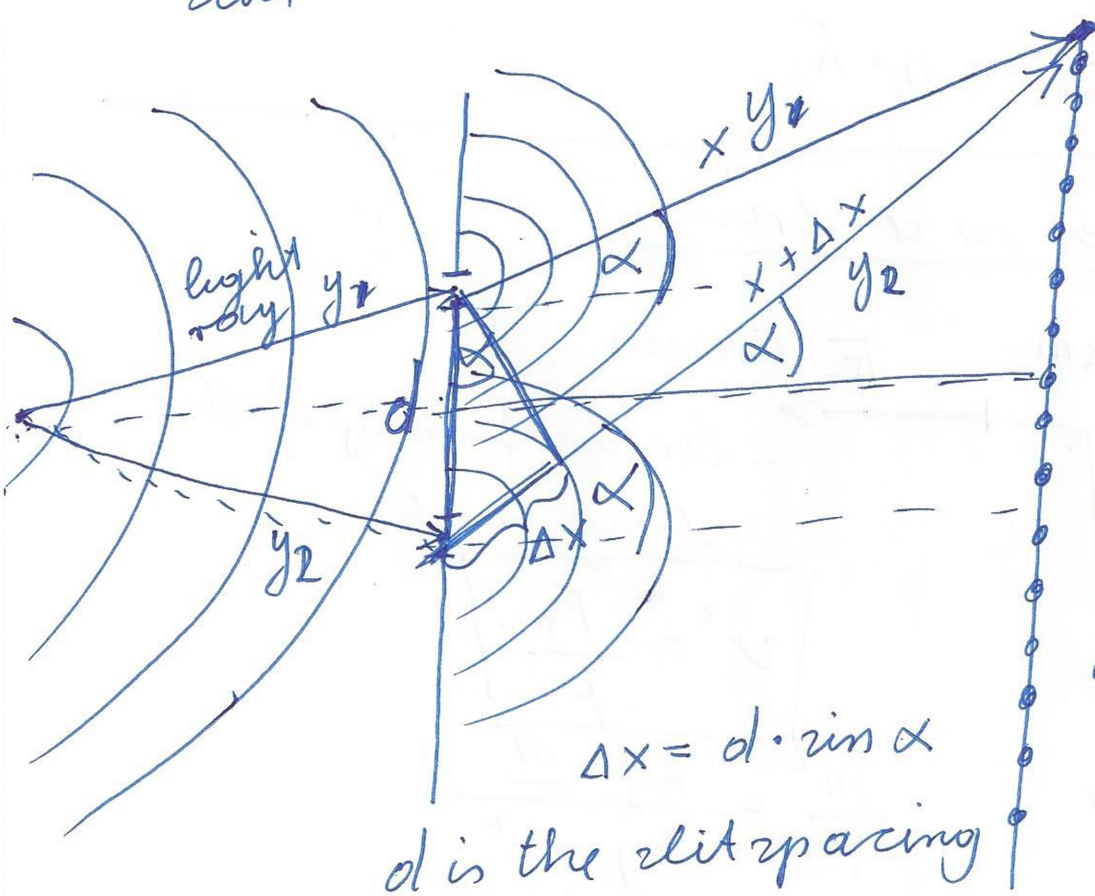
We get maxima whenever $\cos\frac{\beta}{2} = \pm 1$

which means: $\frac{\beta}{2} = n\pi$.

Constructive interference.

Double slit experiment (dse):

We shine a laser beam onto an opaque slide with two narrow slits.



y_2 travels a longer distance by Δx

α is the angle measured from the center between the two slits.

$\Delta x = d \cdot \sin\alpha$

Δx is called the path difference.

d is the slit spacing (distance between the slits).

$$y_1 = A\sin(kx - \omega t)$$
$$y_2 = A\sin(k(x+\Delta x) - \omega t)$$
$$= A\sin(kx - \omega t + k\Delta x)$$

$k\,\Delta x$ is the phase difference β

$$\beta = k \cdot \Delta x$$

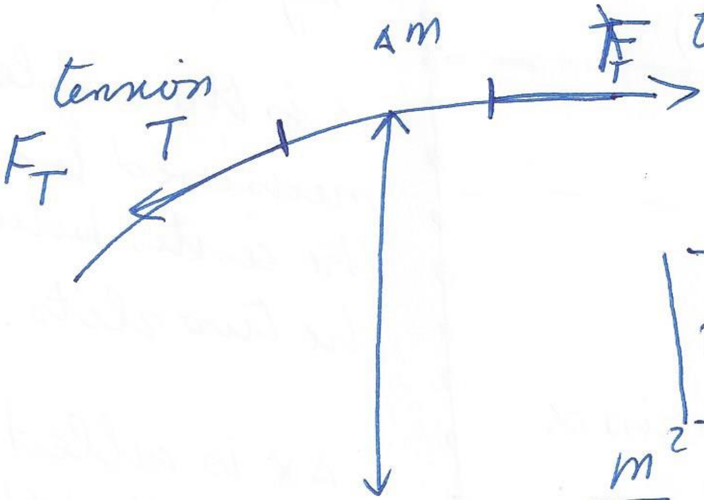
We have a maximum when $\frac{\beta}{2} = n \cdot \pi$

$$\frac{k \cdot \Delta x}{2} = n \cdot \pi$$

$$\frac{\cancel{2}\cancel{\pi}}{\lambda} \cdot \frac{\Delta x}{\cancel{2}} = n \cancel{\pi}$$

$$\Delta x = n \cdot \lambda$$

Standing waves on a string:



linear density of the string is μ

$$\boxed{v^2 = \frac{F_T}{\mu}}$$

$$\frac{m^2}{s^2} = \frac{kg \cdot \frac{m}{s^2}}{\frac{kg}{m}}$$