Lect: 19 A1 - A 1 - Review lecture after absence of 3 weeks ($2\frac{1}{2}$)

a) - Static Equilibrium

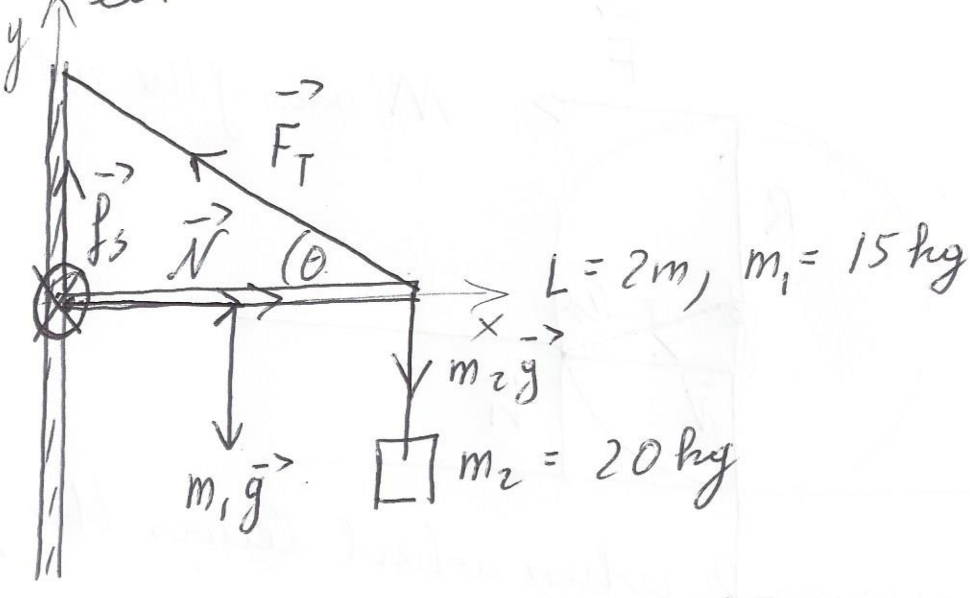
b) - Fluids

c) - Oscillations

d) - Waves

a) $\sum \vec{F}_{ext} = \vec{0}$

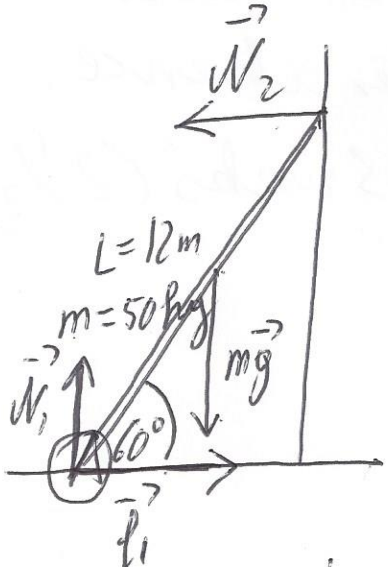
$\sum \vec{\tau}_{ext} = \vec{0}$ $\qquad \vec{\tau} = \vec{r} \times \vec{F}$



$$\begin{cases} \sum F_x = 0 & N - F_T \cos\theta = 0 \\ \sum F_y = 0 & f_s - m_1 g - m_2 g + F_T \sin\theta = 0 \end{cases}$$

$$1\cancel{m} \cdot m_1 g + 2\cancel{m} \cdot m_2 g - F_T \sin\theta \cdot 2 = 0$$

$$f_s = \mu_s \cdot N$$



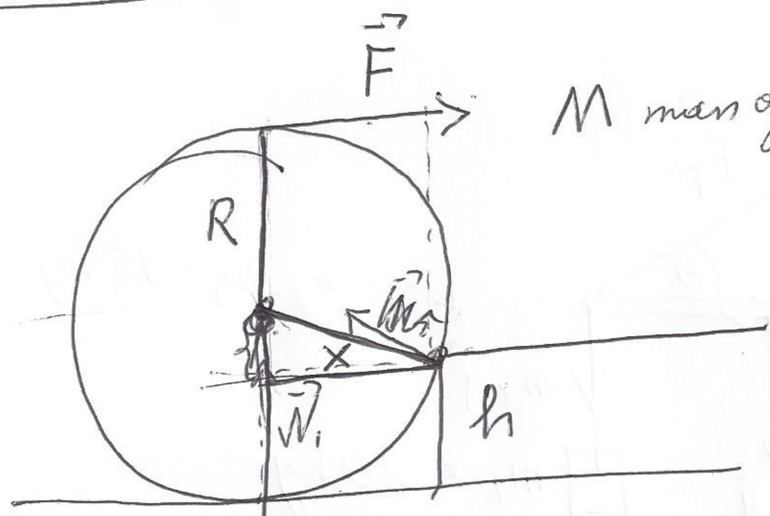
assume that the wall is friction less

$x: f_1 - N_2 = 0$

$y: N_1 - mg = 0$

$$\frac{L}{2} \cdot \cos 60° \cdot mg - N_2 \cdot L \sin 60° = 0$$

$$f_1 = \mu_s \cdot N_1 = \mu_s \cdot mg$$



M mass of the wheel

N_1 becomes 0 when wheel leaves the ground.

$M\vec{g}$

torque by $\vec{F}$ must equal torque by $M\vec{g}$

torque by $\vec{F}$: $\quad F \cdot (R + R - h) = (2R - h) \cdot F$

torque by $M\vec{g}$: $\quad R^2 = x^2 + (R-h)^2; \quad x = \sqrt{R^2 - (R-h)^2}$

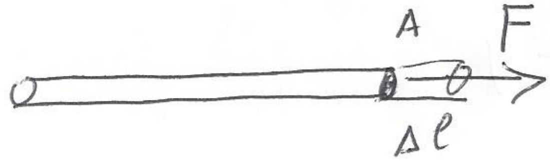
$$(2R - h)F = mg\sqrt{R^2 - (R-h)^2}$$

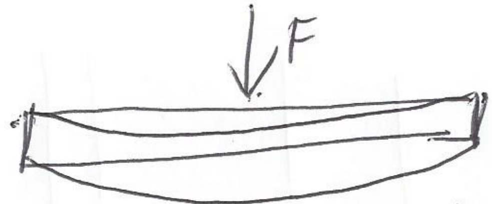
elastic deformations

$$\frac{F}{A} = Y \frac{\Delta l}{l}$$

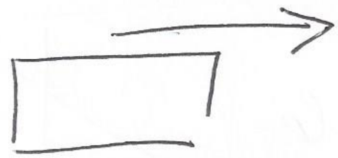
$Y = E$ elastic modulus

Young's modulus





elastic → permanent deformation → break

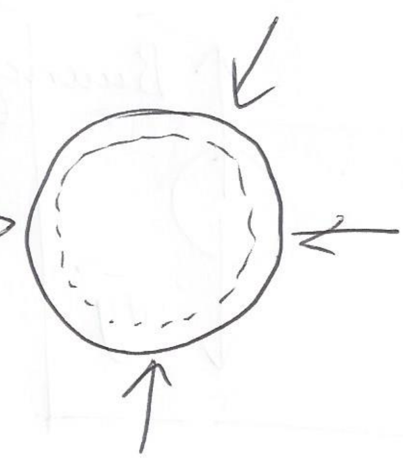


shear stress

compression stress.

volume stress:

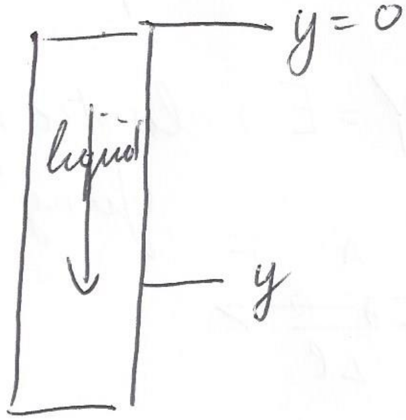
$$\boxed{\frac{F}{A} = -B \frac{\Delta V}{V}}$$



bulk modulus

b) Pascal's principle

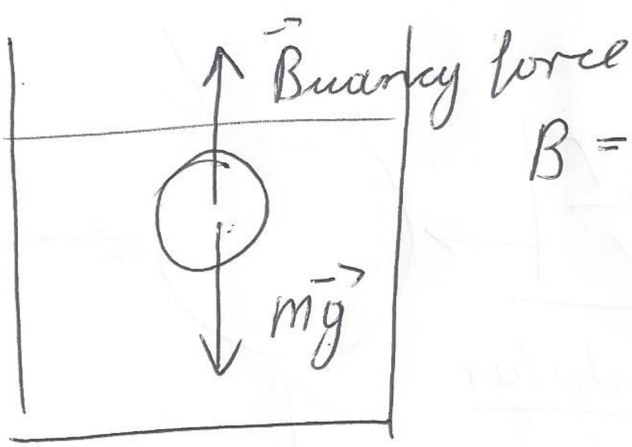
$\Delta P = \rho g \Delta y$



ρ = density

at the same height
= same pressure.

Archimedes principle



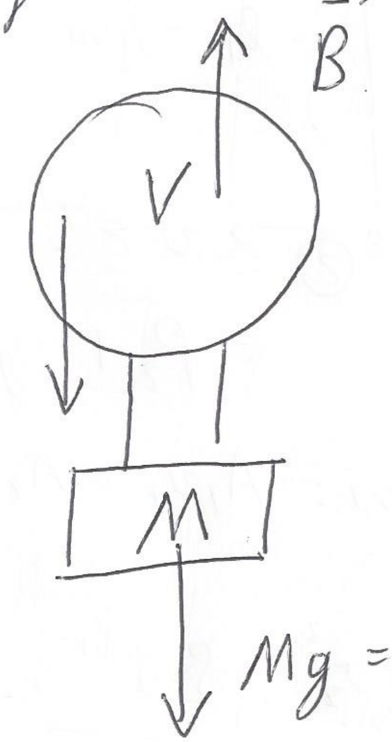
B = weight of the displaced liquid.

Hot air balloon: rises to a height of 5000 m where the air density is ρ_{air} $1.013 \cdot 10^5$ Pa.

Balloon is filled with Helium with density ρ_{He}

mass of balloon skin + cargo $= M$

What volume of the balloon is necessary?



W_{He} = weight of Helium

Mg = weight or cargo

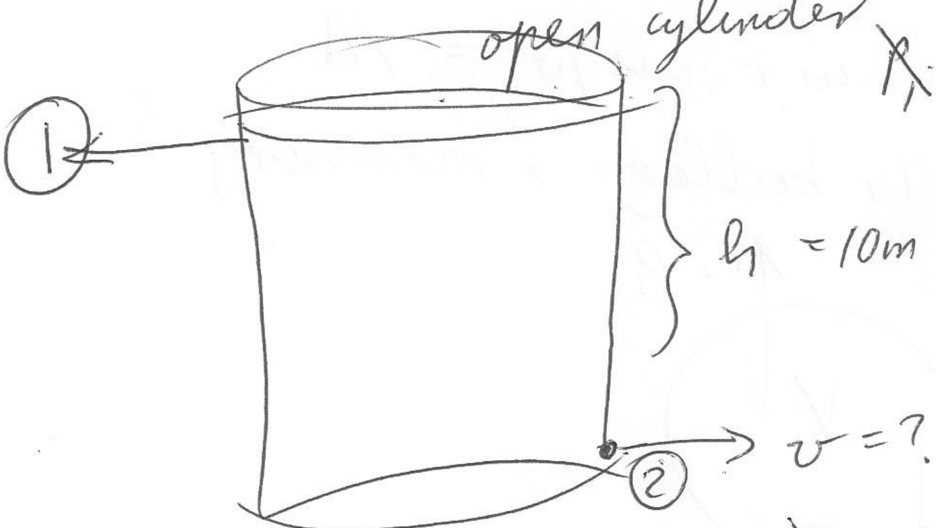
$$B = W_{He} + Mg$$

$$\rho_{air} \cdot V \cdot \cancel{g} = \rho_{He} \cdot V \cancel{g} + M \cancel{g}$$

$$V = \frac{M}{\rho_{air} - \rho_{He}}$$

Bernoulli's principle :

$$P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant.}$$



$$\cancel{P_1} + \rho g y_1 + \frac{1}{2}\rho \cancel{v_1^2}$$

$$\rho g \cdot h$$

$$= \cancel{P_2} + \cancel{\rho g y_2} + \frac{1}{2}\rho v_2^2$$

Continuity equation: $A_1 v_1 = A_2 v_2$

$$\frac{1}{2}\cancel{\rho} v_2^2 = \cancel{\rho} g h$$

$$v_2 = \sqrt{2gh}$$

Oscillations:

$$F = -kx = m\ddot{x}$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$x(t) = x_0 \cos(\omega t + \phi)$$ general solution

$$\dot{x} = -x_0 \omega \sin(\omega t + \phi)$$

$$\ddot{x} = -x_0 \omega^2 \cos(\omega t + \phi)$$

$$-\cancel{x_0}\omega^2 \cancel{\cos(\omega t + \phi)} = -\frac{k}{m}\cancel{x_0}\cancel{\cos(\omega t + \phi)}$$

$$\boxed{\omega^2 = \frac{k}{m} = \left(\frac{2\pi}{T}\right)^2}$$

Standard conditions $x(0) = A$
$v(0) = 0$

$$\boxed{x = A\cos\omega t} \quad \omega_0 = \sqrt{\frac{k}{m}}$$

resonance frequency.

dampening b: $F = -b \cdot v$;

$$x = x_0 e^{-\frac{b}{2m}t}\cos(\omega_1 t + \phi)$$

$$\omega_1 = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

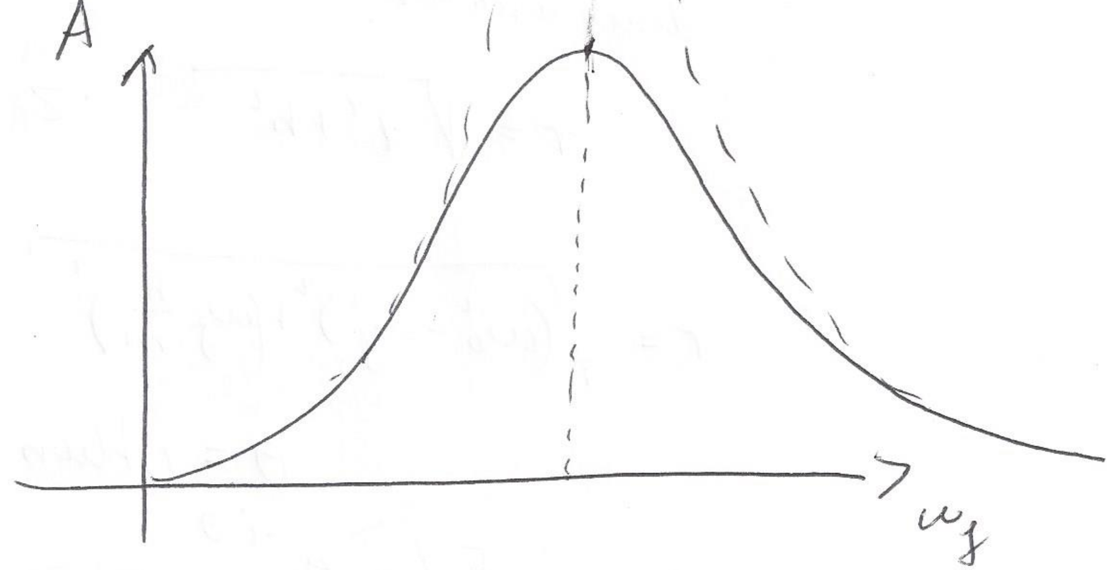
$$m\ddot{x} + kx = F_0 \cos\omega_g t$$

try: $x = A\cos\omega_g t$

$$\ddot{x} + \omega_0^2 x = \frac{F_0}{m}\cos\omega_g t$$

$$-A\omega_g^2 \cancel{\cos\omega_g t} + \omega_0^2 A \cancel{\cos\omega_g t} = \frac{F_0}{m}\cancel{\cos\omega_g t}$$

$$A = \frac{F_0/m}{\omega_0^2 - \omega_g^2}$$



With dampening

$$m\ddot{x} + b\dot{x} + kx = F_0\cos\omega_g t$$

We use $\hat{F} = F_0 e^{i\omega_g t}$ find real solution

try: $\hat{x} = A e^{i\omega_g t}$

$$A\cdot(i\omega_g)^2\cdot e^{i\omega_g t} + \frac{b}{m} i\omega_g A e^{i\omega_g t} + \omega_0^2 A e^{i\omega_g t} = F_0/m\, e^{i\omega_g t}$$

$$-\omega_g^2 A + i\omega_g \frac{b}{m} A + \omega_0^2 A = F_0/m$$

$$\hat{A} = \frac{F_0/m}{\omega_0^2 - \omega_g^2 + i\omega_g \frac{b}{m}}$$

write in exponential form. $r\cdot e^{i\theta}$

↑

denominator

$$r = \sqrt{a^2+b^2} \qquad \hat{z} = a+ib$$

$$r = \sqrt{(\omega_0^2-\omega_g^2)^2+\left(\omega_g \frac{b}{m}\right)^2}$$

$$\theta = \text{Atan}\,\frac{b}{a}$$

$$\hat{A} = \frac{F_0/m\, e^{-i\theta}}{\sqrt{(\omega_0^2-\omega_g^2)+\left(\omega_g \frac{b}{m}\right)^2}}$$

$$x = \hat{A} e^{i\omega_g t}$$
$$= |A|\, e^{i(\omega_g t-\theta)}$$