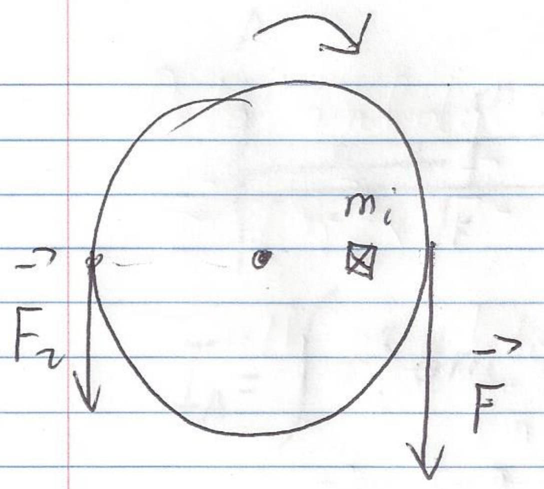
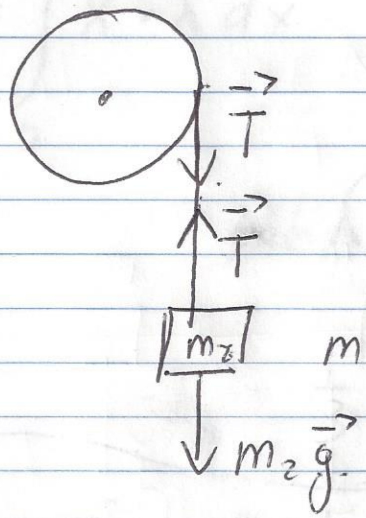


$$\sum \vec{\tau}_{ext} = I_A \vec{\alpha}$$

$$I_A = \sum m_i r_i^2$$
$$= \int_{object} r^2 \, dm$$

Ex.



$M = 3kg$
$r = 10\,cm$

$$I_{CM} = \frac{1}{2} M R^2$$

$m_2 = 0.5kg$

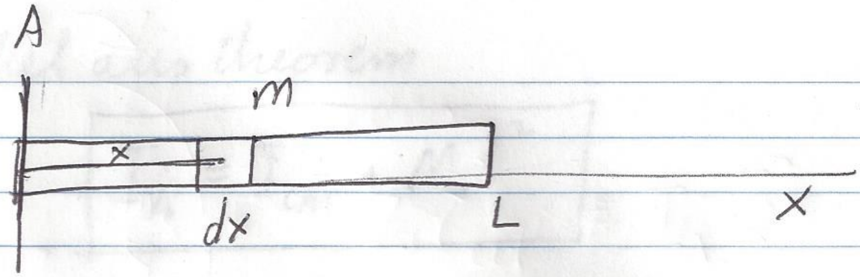
$$m_2 g - T = m_2 a \qquad a = \alpha \cdot r$$

$$T \cdot r = I_{CM} \cdot \alpha = \frac{1}{2} M r^2 \cdot \alpha$$

$$T \cdot \cancel{r} = \frac{1}{2} M \cancel{r} a$$

$$m_2 g - \frac{1}{2} M a = m_2 a$$

$$m_2 g = a \left(m_2 + \frac{1}{2} M \right), \quad a = \frac{m_2 g}{m_2 + \frac{1}{2} M}$$



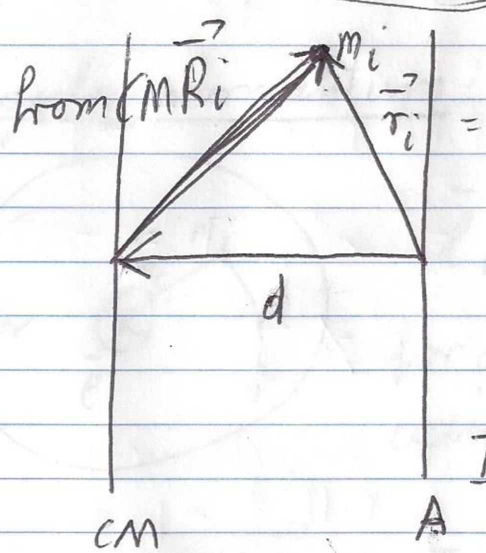
$$I_A = \int r^2\, dm \qquad dm = \lambda\, dx$$

λ = linear density

$$I_A = \int_0^L x^2 \cdot \lambda\, dx = \frac{x^3}{3}\lambda \Big|_0^L = \frac{L^3}{3}\lambda$$

$$\cancel{m} L \cdot \lambda = m$$

$$I_A = \frac{1}{3} m L^2 \qquad I_A = \sum m_i \vec{r}_i^{\,2}$$



$\vec{r}_i$ = from A

$$\vec{r}_i = \vec{d} + \vec{R}_i$$

$$I_A = \sum m_i (\vec{d} + \vec{R}_i)^2$$

$$= \sum m_i (\vec{d}^{\,2} + \vec{R}_i^{\,2} + 2\vec{d} \cdot \vec{R}_i)$$

$$I_A = Md^2 + I_{CM} + 2\vec{d}\, \underbrace{\cancel{\sum m_i \vec{R}_i}}_{0}$$

Parallel axis theorem

$$\boxed{I_A = I_{cm} + Md^2}$$

$$I_A = \frac{1}{3} ML^2$$

$$I_{CM} = \frac{1}{3} ML^2 - Md^2 \qquad d = \frac{L}{2}$$

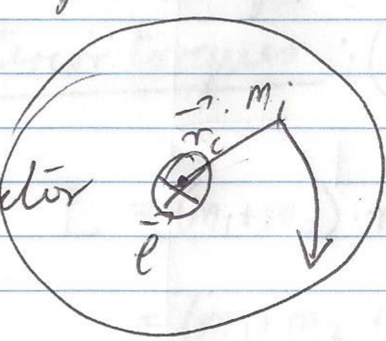
$$= \frac{1}{3} ML^2 - M\frac{L^2}{4}$$

$$I_{CM} = \frac{1}{12} ML^2$$

$$I_{CM} \text{ sphere} = \frac{2}{5} MR^2$$

<u>Angular momentum</u>: $\vec{l} = \vec{r} \times \vec{p}$

$\vec{k}$ is unit vector $\perp$ paper



$$l_i = \vec{r}_i \times \vec{p}_i$$

for a rotation around a fixed axis: $\vec{r}_i \perp \vec{p}_i$

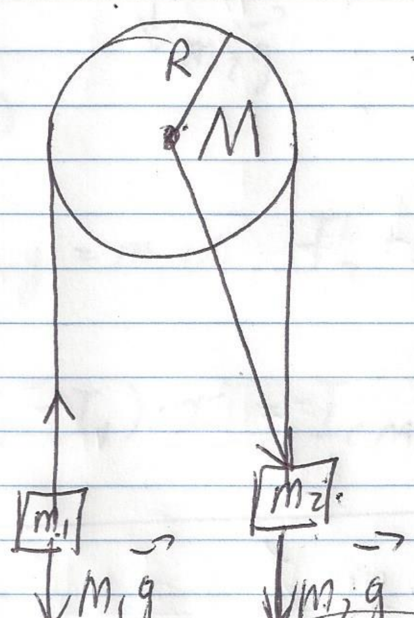
$$\sum \vec{l}_i = \sum \vec{r}_i \times \vec{p}_i = L \cdot \vec{k}$$

$$L = \sum r_i p_i = \sum r_i m_i v_i = \sum m_i r_i^2 \omega$$

$$\vec{L}_A = I_A \vec{\omega} \qquad I = \sum m_i r_i^2$$

linear : $\sum \vec{F}_{ext} = m\vec{a} = \frac{d\vec{p}}{dt}$

rotation $\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$



$m_2 = 1.2\ kg$

$m_1 = 1.0\ kg$

$M = 10\ kg \quad R = 10\ cm$

disk.

$\vec{l}_2 = \vec{r}_2 \times \vec{p}_2$

$l_2 = r_2 \cdot m_2 \cdot v$

$l_1 = r_1 \cdot m_1 \cdot v$

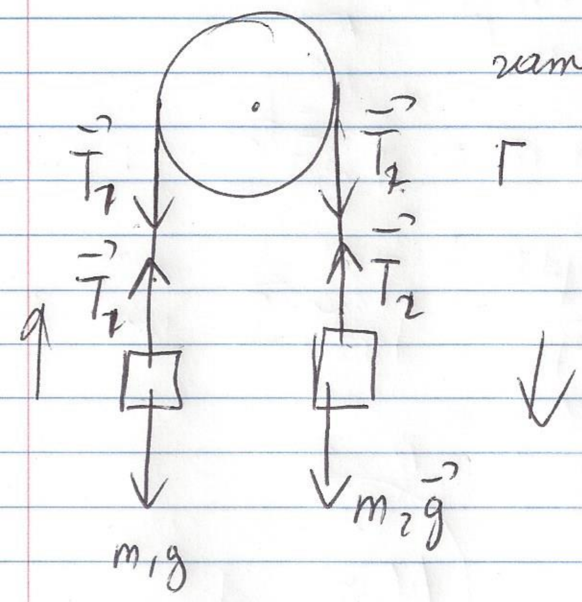
$L_{disk} = I_{cm} \cdot \omega$

Exterior torques : $(m_2 - m_1)\, g \cdot R$

$$L = (m_1 + m_2) \cdot r^2 \omega + \frac{1}{2} M r^2 \omega$$

$$= (m_1 + m_2 + \frac{1}{2} M)\, \underbrace{r^2 \omega}_{r \cdot v}$$

$$\frac{dL}{dt} = \left(m_1 + m_2 + \frac{M}{2}\right) \cdot r \cdot a$$



same result as applying

Γ

$$T_1 - m_1 g = m_1 a \quad -T_2 + m_2 g = m_2 a$$

$$(T_2 - T_1) \cdot r = I_{cm} \cdot \alpha$$
