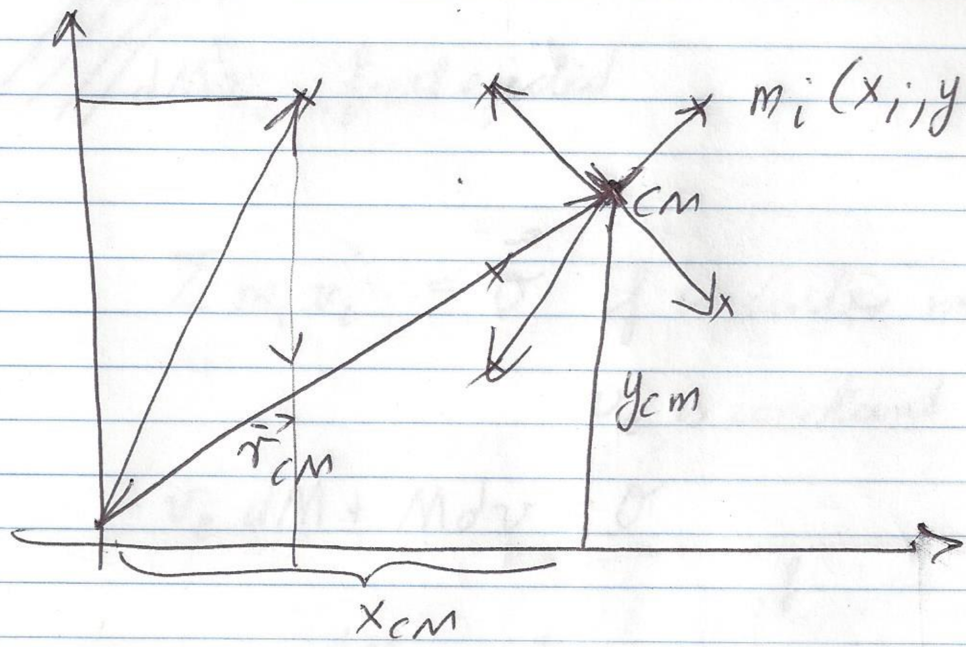
Point masses $\longrightarrow$ extended objects

Center of mass of n objects considered as point masses:

$$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i} ; \qquad \sum m_i = M$$

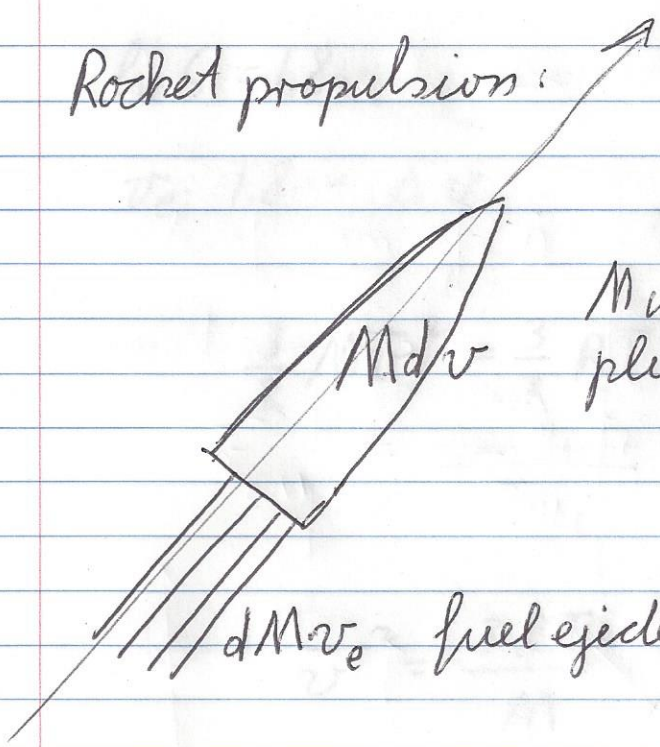


$$M \vec{r}_{CM} = \sum m_i \vec{r}_i$$

$$\frac{d}{dt} \rightarrow M \vec{v}_{CM} = \sum m_i \vec{v}_i \quad \|$$

$$\frac{d}{dt} \rightarrow M \vec{a}_{CM} = \sum m_i \vec{a}_i = \sum \vec{F}_i$$

Rocket propulsion:



M includes the mass of the hull plus the mass of the fuel
H_2+O_2

$$\sum m_i \vec{v}_i = \vec{0}$$ if in center mass

v_e is constant

$$-v_e\, dM + M\,dv = 0$$

$$v_e \frac{dM}{M} = dv \quad \Big|_i^f$$

interpret dM as $-|dM|$

$$-v_e \frac{|dM|}{M} = dv$$

$$-v_e \ln \frac{M_f}{M_i} = v_f - v_i$$

$$v_e \ln \frac{M_i}{M_f} = v_f - v_i \qquad \frac{M_i}{M_f} = 6$$

$\ln 6 = 1.8$

$v_e \cdot 1.8 = \Delta v$

$$\frac{1}{2} M\bar{v}^2 = \frac{3}{2} RT \quad ; \quad M_{water} = 18g = 0.018kg$$

$T \approx 2000K$

$$\bar{v}^2 = \frac{3RT}{M} \quad , \quad \bar{v} \approx \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3 \cdot 8.314 \cdot 2000}{0.018}} \approx 1.7 \frac{km}{s}$$

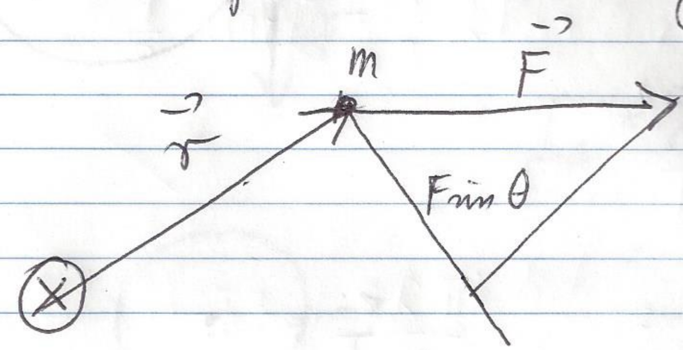
$$\Delta v \approx 1.7 \cdot 10^3 \cdot 1.8 = 3000 \frac{m}{s}$$

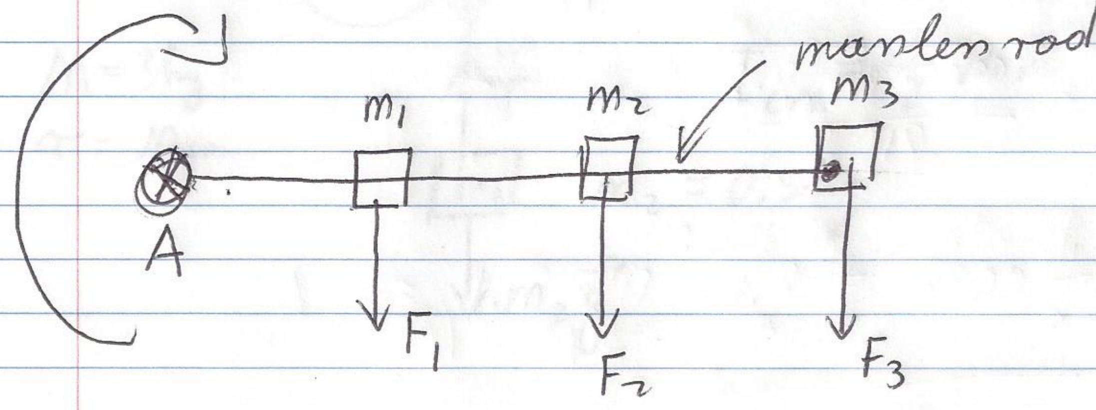
$$v_{esc} = 1.4 \sqrt{9.8 \cdot 6.37 \cdot 10^6} = 1.1 \cdot 10^4 \frac{m}{s}$$

Rotational dynamics

$\vec{\tau} = \vec{r} \times \vec{F}$; $\tau = r \cdot F \cdot \sin\theta = r_\perp F = r F_T$

with repect to a center of rotation.





$\tau_1 + \tau_2 + \tau_3 =$

$= r_1 F_1 + r_2 F_2 + r_3 F_3$ $\qquad F = ma$

$= r_1 m_1 a_1 + r_2 m_2 a_2 + r_3 m_3 a_3$

$= r_1 m_1 \alpha r_1 + r_2 m_2 \alpha r_2 + r_3 m_3 \alpha r_3$

$= \left(\sum m_i r_i^2\right) \alpha = I_A \cdot \alpha$