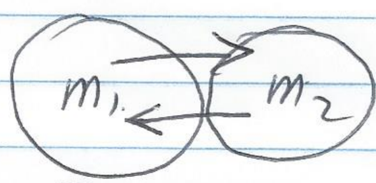
Lekture 15 Momentum 2

$\vec{p} = \gamma m_0 \vec{v}$; $\quad d\vec{J}$ impulse $= d\vec{p} = \vec{F} \cdot dt$

$$\vec{J} = \vec{p}_2 - \vec{p}_1$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

<u>Collisions</u>: As all forces (essential) during a collision are internal, momentum is conserved.



$$\vec{F}_1 + \vec{F}_2 = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$$

$$d\vec{p}_1 + d\vec{p}_2 = \vec{0}$$

$$\Delta(\vec{p}_1 + \vec{p}_2) = \Delta\vec{p}_1 + \Delta\vec{p}_2 = \text{constant}$$

total momentum is conserved

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

- inelastic heat gets "lost"

- elastic collisions: energy conservation in addition to momentum conservation

$v_1 = 40\,\frac{m}{s}$ $m_1 = 3\,kg$ $v_2 = 50\,\frac{m}{s} \leftarrow$ $m_2 = 2\,kg$

1) $p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$

$40 \cdot 3 + (-50) \cdot 2 = \cancel{40}\, 3 v_{1fx} + 2 v_{2fx}$

$20 = 3 v_{1f} + 2 v_{2f}$

elastic collision

$K_{1i} + K_{2i} = K_{1f} + K_{2f}$

2) $\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

In 1) and 2) bring m_1 terms to left and m_2 terms to the right:

$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i})$

$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_{2i}^2)$

$v_{1i} + v_{1f} = v_{2f} + v_{2i}$

$v_{1i} - v_{2i} = v_{2f} - v_{1f}$; $\Delta v_{in} = -\Delta v_f$

$$20 = 3v_{1f} + 2v_{2f}$$

$$\Delta v_i = -\Delta v_f$$

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

$$40 + 50 = v_{2f} - v_{1f}$$

$$+90 = v_{2f} - v_{1f}$$

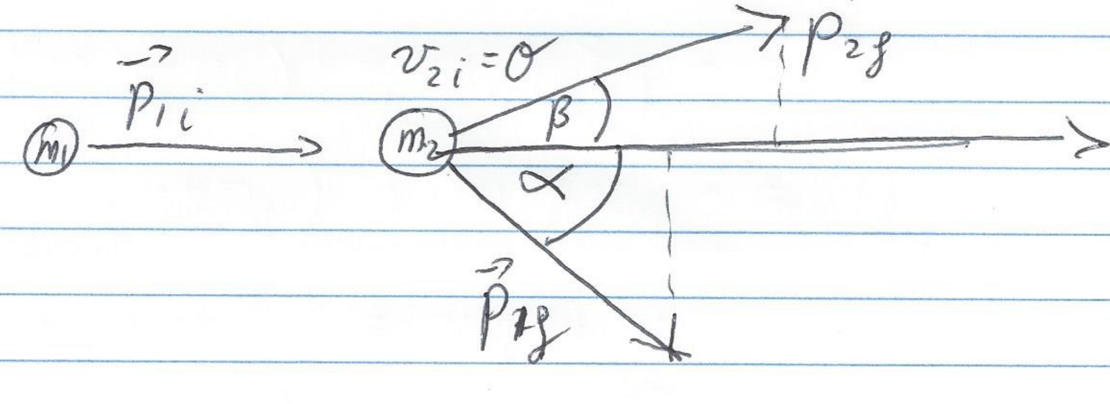
$$v_{2f} = v_{1f} + 90$$

$$20 = 3v_{1f} + 2v_{1f} + 180$$

$$-160 = 5v_{1f} \qquad v_{1f} = -\overset{32}{\cancel{17}}\ \frac{m}{s}$$

$$v_{2f} = -32\frac{m}{s} + 90\frac{m}{s} = 58\frac{m}{s}$$

Two dimensional: elastic collision.



$$\vec{p}_{1i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$p_{1ix} = p_{1fx} + p_{2fx}$$

1) $$p_{1ix} = p_{1f} \cdot \cos\alpha + p_{2f} \cos\beta$$

2) $$p_{1iy} = p_{1fy} + p_{2fy}$$

$$0 = -p_{1f} \sin\alpha + p_{2f} \sin\beta$$

Prove that $\alpha + \beta = \frac{\pi}{2}$ for an elastic collision!

3) $$\cancel{\tfrac{1}{2}} m_1 v_{1i}^2 = \cancel{\tfrac{1}{2}} m_1 v_{1f}^2 + \cancel{\tfrac{1}{2}} m_2 v_{2f}^2$$

$$m_1 \vec{v}_{1i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad \Big|^2$$

$$m_1^2 v_{1i}^2 = m_1^2 \vec{v}_{1f}^2 + m_2^2 v_{2f}^2 + 2 m_1 m_2 \vec{v}_{1f} \cdot \vec{v}_{2f}$$

$$= m_1^2 \vec{v}_{1f}^2 + m_2^2 \vec{v}_{2f}^2 + 2 m_1 m_2 \vec{v}_{1f} \vec{v}_{2f}$$

If $m_1 = m_2$?

Cancel all masses

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2$$

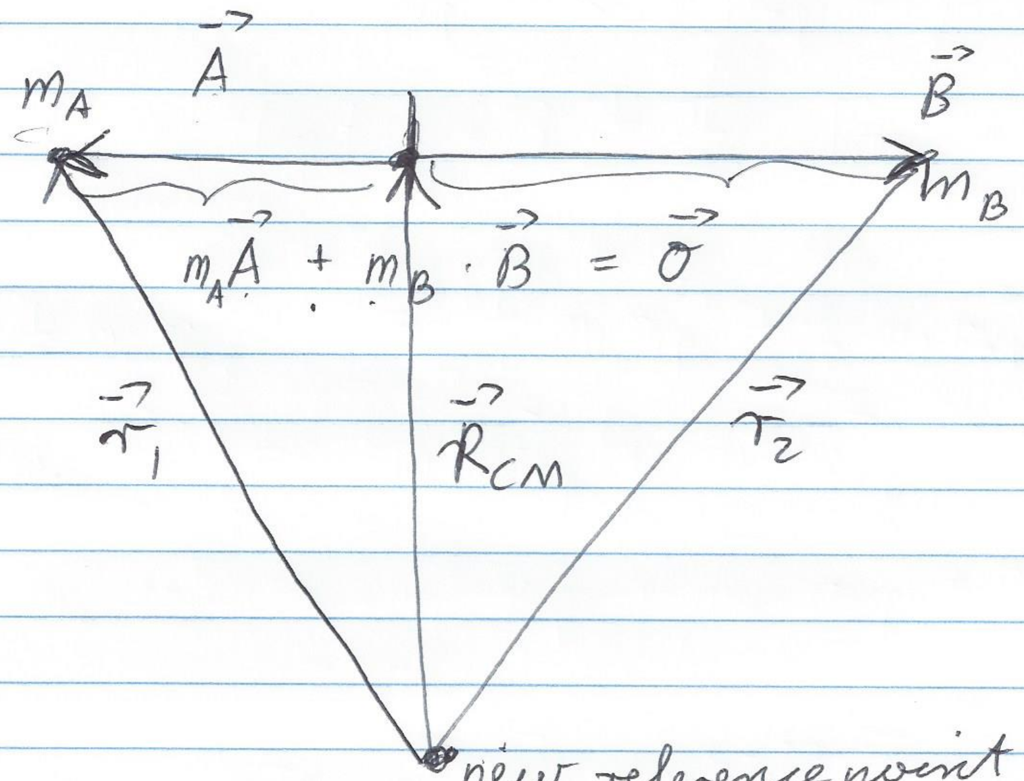
$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + 2 \vec{v}_{1f} \cdot \vec{v}_{2f}$$

$$\vec{v}_{1f} \cdot \vec{v}_{2f} = 0 \implies \alpha + \beta = \frac{\pi}{2}$$

$$v_{1f} \cdot v_{2f} \cdot \cos\theta = 0$$

$$\cos\theta = 0 \qquad \theta = \frac{\pi}{2}, \text{ or } \frac{3\pi}{2}$$

Center of mass



$$\vec{R}_{CM} = \vec{r}_1 + \vec{A} \qquad \vec{R}_{CM} = \vec{r}_2 + \vec{B}$$

$$\vec{A} = \vec{R}_{CM} - \vec{r}_1 \qquad \vec{B} = \vec{R}_{CM} - \vec{r}_2$$

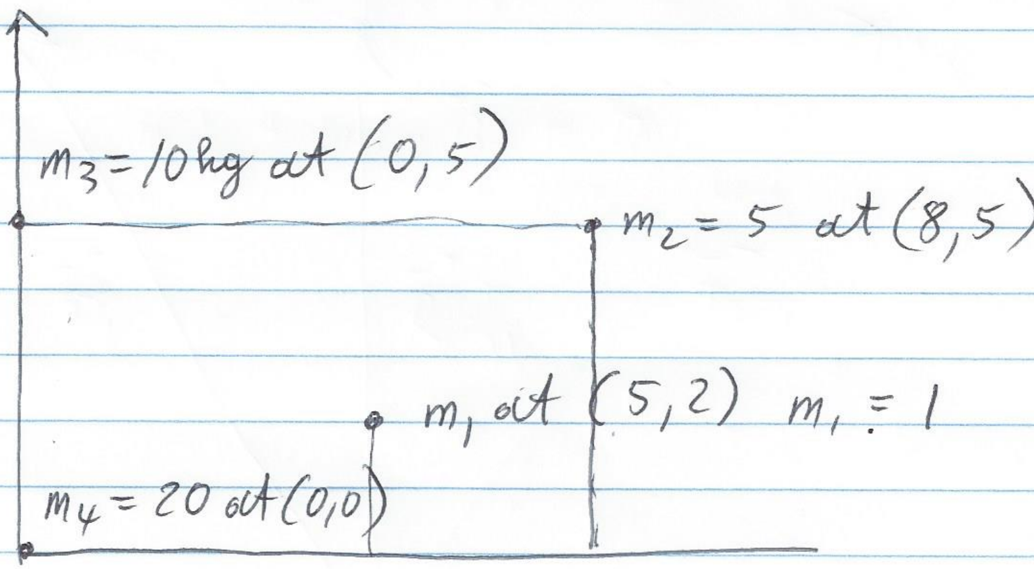
$$m_A (\vec{R}_{CM} - \vec{r}_1) + m_B (\vec{R}_{CM} - \vec{r}_2)$$

$$(m_A + m_B) \vec{R}_{CM} = m_A \vec{r}_1 + m_B \vec{r}_2$$

$$\vec{R}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$\boxed{\vec{R}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}}$$

$$\boxed{\begin{aligned} x_{CM} &= \frac{\sum m_i x_i}{\sum m_i} \\ y_{CM} &= \frac{\sum m_i y_i}{\sum m_i} \end{aligned}}$$



$$x_{CM} = \frac{5\cdot 1 + 5\cdot 8 + 0 + 0}{36} \qquad \sum m_i = 36 \text{ kg}$$

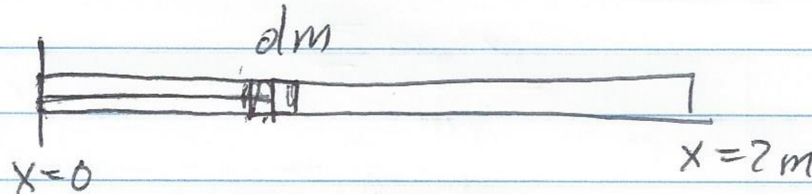
$$= \frac{45}{36} \text{ m}$$

$$y_{CM} = \frac{2\cdot 1 + 5\cdot 5 + 5\cdot 10 + 0}{36}$$

$$\vec{r}_{CM} = \frac{\int_{object} \vec{r} \cdot dm}{\int_{object} dm}$$

$$dm = \lambda \cdot dx$$

λ = linear mass density in $\frac{kg}{m}$



$$\int x \, dm$$

$$\lambda = \left(\frac{2}{m} x + 1\right) \frac{kg}{m}$$

$$= \frac{2\,kg}{m^2} x + 1 \frac{kg}{m}$$

$$\int_0^2 dm = \int_0^2 \lambda \, dx = \int_0^2 (2x+1)\, dx$$

$$= \left(\frac{2x^2}{2} + x\right)\Bigg|_0^2 = 4 + 2 = 6\,kg$$

$$\int_0^2 x \, dm = \int_0^2 x \lambda \, dx = \int_0^2 x(2x+1)\, dx$$

$$x_{CM} = 1.22\,m \qquad = \int_0^2 (2x^2 + x)\, dx = \left(\frac{2x^3}{3} + \frac{x^2}{2}\right)\Bigg|_0^2$$

$$\frac{16}{3} + 2 = 7.333\,kg\,m$$

For a surface density we have

$$dm = \sigma \cdot dA$$

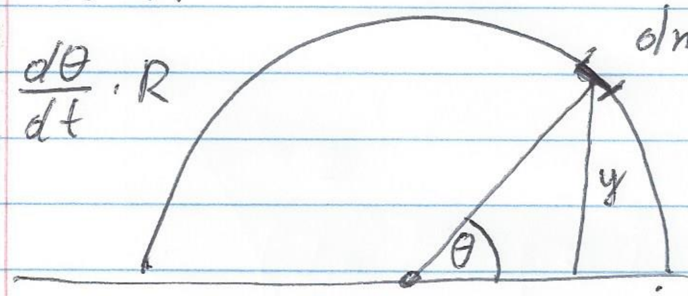
σ = surface density in $\frac{kg}{m^2}$

$dA = dx \cdot dy$ in Cartesian coordinates

$r\,dr\,d\theta$ in polar coordinates.

$v_\theta = \omega \cdot R$

$\frac{ds}{dt} = \frac{d\theta}{dt} \cdot R$



$dm = ds \cdot \lambda$

wire bent into a semi circle, with uniform linear density λ

$x_{CM} = 0$

$$y_{CM} = \frac{\int y\,dm}{\int dm} \; ; \quad \int dm = \pi R \lambda$$

$$y = R \sin\theta$$

$$\int R \sin\theta \, \lambda \cdot R\,d\theta = \int_0^\pi \sin\theta \, d\theta \cdot R^2 \lambda$$

$$= R^2 \lambda \underbrace{(-\cos\theta)\Big|_0^\pi}_{=2} = 2R^2\lambda$$

$$y_{CM} = \frac{2R^{\cancel{2}}\cancel{\lambda}}{\pi \cancel{R}\cancel{\lambda}} = \frac{2R}{\pi}$$