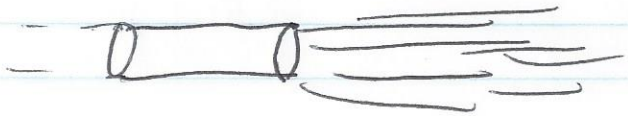
Lecture 14 Momentum $\vec{p}$

$$\vec{p}=m\vec{v} \quad \left[kg\cdot\frac{m}{s}\right]$$

$$\vec{F}=\frac{d\vec{p}}{dt}=\frac{dm}{dt}\vec{v}+m\cdot\frac{d\vec{v}}{dt}$$



$$\frac{dm}{dt}=10\,kg/s$$

$$v=20\frac{m}{s}$$

$$\vec{F}=\frac{dm}{dt}\vec{v}=10\frac{kg}{s}\cdot 20\frac{m}{s}=\underline{\underline{200N}}$$

relativistic: $\vec{p}=\gamma m_0\vec{v}$ $\quad \gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

reminder $\quad E=\gamma\cdot m_0 c^2$

$$E=\sqrt{p^2c^2+m_0^2c^4}$$

if $m_0=0$ $\quad E=p\cdot c$

Back step to escape velocity in connection with energy conservation.

Potential energy of gravitation

$$U=-\frac{mMG}{r} \quad \vec{F}=-\frac{mMG}{r^2}\vec{U}_r$$

$$\frac{1}{2} m v_1^2 - \frac{mMG}{r_1} = \frac{1}{2} m v_2^2 - \frac{mMG}{r_2}$$

if satellite escapes the attractive force

$$U = 0 \quad \vec{F}_g = 0$$

$$= 0 + 0$$

energy conservation for an escaping satellite or particle.

$$v_1^2 = \frac{2MG}{r_1}$$

$$v_{esc} = \sqrt{\frac{2MG}{r_1}}$$

free satellite

$$v = \sqrt{\frac{MG}{r}} = \sqrt{\frac{MG}{r^2} r} = \sqrt{g \cdot r}$$

on earth or planet:

$$v_{esc} = \sqrt{\frac{2MG}{r_1^2} \cdot r_1} = \sqrt{2 g \cdot r_1}$$

---

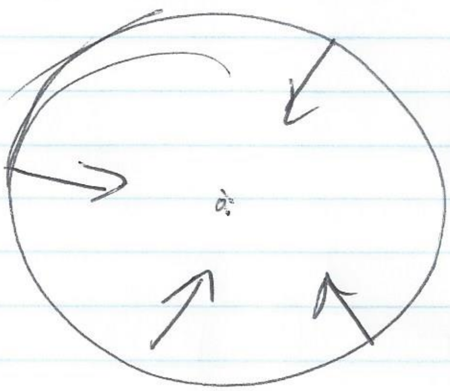
if the energy of the particle is relativistic

$K \rightarrow mc^2$

$$\cancel{m} c^2 - \frac{\cancel{m} MG}{r} = 0 \quad r = \frac{MG}{c^2}$$

$r = \frac{MG}{c^2}$ = Schwarzschild radius

radius of a black hole



if the mass of a sun is large enough it starts to pull the outer layers of mass towards the center.

$\frac{mMG}{r^2}$ M remains the same, r becomes smaller.

$r_{atom} \approx 10^{-10} m$ $r_{nucleus} \approx 10^{-15} m$

supernova

earth: $M = 5.98 \cdot 10^{24}$ $G = 6.673 \cdot 10^{-11}$

$c = 3 \cdot 10^{8}$

$r = 4.4 \cdot 10^{-3} m$

4.4 m

$$\vec{p} = m \cdot \vec{v} \qquad \sum \vec{F} = \cancel{m}\frac{d\vec{p}}{dt}$$

$$\sum \vec{F} = m\vec{a}$$ <u>work energy</u>

$$dW = \vec{F}\cdot d\vec{r} = m\vec{a}\cdot d\vec{r}$$

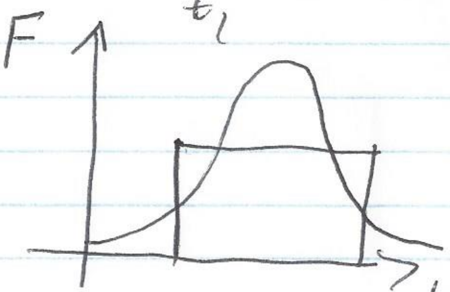
one dimension $dW = F \cdot dx$

$$F = \frac{dW}{dx}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

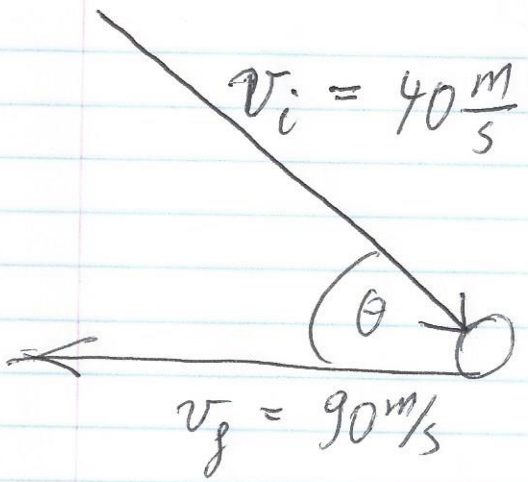
$$\int_{t_2} \vec{F}\cdot dt = \int d\vec{p}$$

$$\int_{t_1} \vec{F}\, dt = \underbrace{\vec{p}_f - \vec{p}_i}_{\text{impulse}} = \Delta\vec{p}$$



$\vec{F}_{average} \cdot \Delta t$

If we can measure Δt and $\Delta\vec{p}$ we can find the average force during a collision.



$\Delta t = 10^{-3} s$

$\vec{F} = ?$

$$\vec{F} = \frac{\vec{\Delta p}}{\Delta t}$$

$$\vec{\Delta p} = \Delta p_x \vec{i} + \Delta p_y \cdot \vec{j}$$
$$= (p_{fx} - p_{ix})\vec{i} + (p_{fy} - p_{iy})\vec{j}$$

$m = 0.5 kg$ $\quad = m(-90 - 40\cos\theta)\vec{i} + m(0 + 40\sin\theta)\vec{j}$

$\theta = 36.9°$

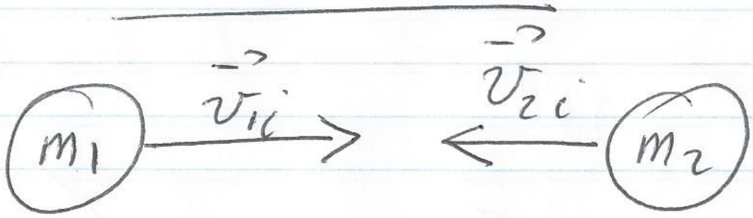
$\sin\theta = 0.6 \quad \cos\theta = 0.8$

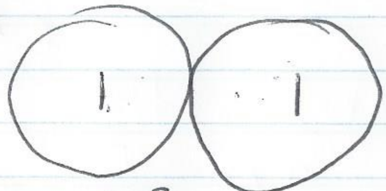
$$\vec{\Delta p} = 0.5(-122)\vec{i} + 0.5(24)\vec{j}$$
$$= -61\vec{i} + 12\vec{j}$$
$$= \langle -61, 12 \rangle \frac{kg\,m}{s}$$

$$\vec{F} = \frac{\vec{\Delta p}}{0.001} = \langle -61000, 12000 \rangle N$$

$$|F| = \sqrt{\cancel{{}^{-}}61000^2 + 12000^2} = 6.22 \cdot 10^4 N$$

Collisions





$$\sum \vec{F}_{ext,1} = \frac{d\vec{p}_1}{dt} \qquad \sum \vec{F}_{ext,2} = \frac{d\vec{p}_2}{dt}$$

$$\underbrace{\sum \vec{F}_{ext,1} + \sum \vec{F}_{ext,2}}_{=0} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt}$$

$$\int d\vec{p}_1 + \int d\vec{p}_2 = \vec{0}$$

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0}$$

$$\vec{p}_{1f} - \vec{p}_{1i} + \vec{p}_{2f} - \vec{p}_{2i} = \vec{0}$$

$$\boxed{\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}}$$

<u>In all collisions total momentum is conserved.</u>

$$\xleftarrow{\quad} \quad v_{2i} = 50\, m/s$$

$$m_1 = 3\,kg \longrightarrow \qquad m_2 = 2\,kg$$

$$v_{1i} = 40\, m/s$$

$$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$$

$$3 \cdot 40 - 2 \cdot 50 = 3 v_{1f} + 2 \cdot v_{2f}$$

Assume the two objects stick together

$$= 5 v_f$$

$$v_f = \frac{120 - 100}{5} = 4 \frac{m}{s}$$

For inelastic collisions energy is lost due to friction, sound, heat.

Elastic collisions: mechanical energy is conserved in addition to momentum conservation.

$$\begin{cases} \vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \\ \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \\ + U_i \qquad\qquad U_f \end{cases}$$

Head on elastic collisions

$$m_1 v_{1xi} + m_2 v_{2xi} = m_1 v_{1xf} + m_2 v_{2xf}$$

$$\cancel{\tfrac{1}{2}} m_1 v_{1i}^2 + \cancel{\tfrac{1}{2}} m_2 v_{2i}^2 = \cancel{\tfrac{1}{2}} m_1 v_{1f}^2 + \cancel{\tfrac{1}{2}} m_2 v_{2f}^2$$

data from previous example

$$3 \cdot 40 - 2 \cdot 50 = 3 v_{1f} + 2 v_{2f}$$

$$20 = 3 v_{1f} + 2 v_{2f}$$

$$3 \cdot 40^2 + 2 \cdot 50^2 = 3 v_{1f}^2 + 2 \cdot v_{2f}^2$$

$$8200 = 3 v_{1f}^2 + 2 v_{2f}^2$$