<u>Energy</u>: Lect 12

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \Delta K \; ; \; K = \frac{1}{2} m \vec{v}^{\,2}$$

non-relativistic

$\vec{F}$ is the net force of (exterior)

We divide $\vec{F}$ into non-conservative forces $\vec{F}_{nc}$ and conservative forces $\vec{F}_c$

All conservative forces have a potential energy associated with them: $U(r)$

$U(x,y,z)$ is a scalar.

$$\vec{F}_c = -\overrightarrow{grad}\, U = -\vec{\nabla} \cdot U$$

$$= -\left\langle \frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right\rangle$$

this leads the work done by a conservative force:

$$W_c = -\Delta U$$

$$W = \int \vec{F}_c \cdot d\vec{r} + \int \vec{F}_{nc} \cdot d r = \Delta K$$

$$= -\Delta U + W_{nc} = \Delta K$$

$$\boxed{W_{nc} = \Delta K + \Delta U}$$

If all forces are conservative

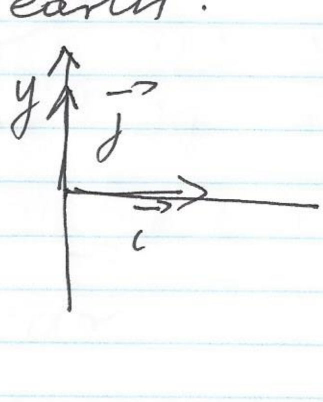
$$\boxed{0 = \Delta K + \Delta U}$$ Conservation of mechanical energy

$$\boxed{K_1 + U_1 = K_2 + U_2}$$

grav on the surface of the earth:

$$\vec{F}_g = -mg\vec{j}$$

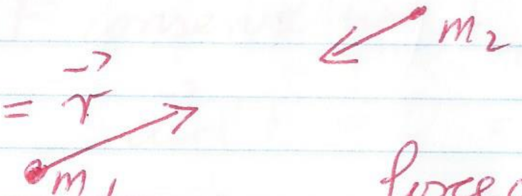
$$U = mgy$$



Spring: $\vec{F}_s = -kx\vec{i}$

$$U = \frac{1}{2}kx^2$$

$r \cdot \vec{u}_r = \vec{r}$

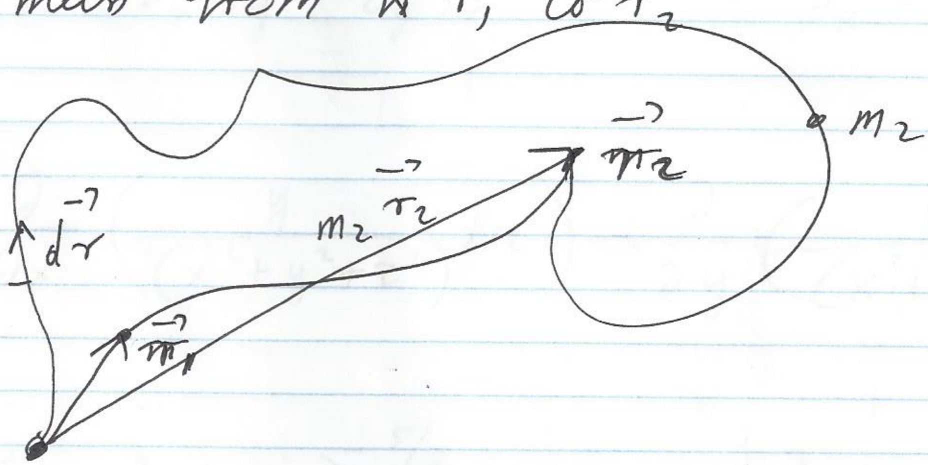


force of m_1 on m_2

$$\vec{F} = -\frac{m_1 m_2 G}{r^2}\vec{u}_r$$

$$= \frac{-m_1 m_2 G}{(x^2+y^2)^{3/2}} \cdot \langle x, y \rangle \quad \text{in two dim}$$

Find the work done by this force for moving a mass from $\vec{r}_1$ to $\vec{r}_2$



$$W = \int -\frac{m_1 m_2 G}{r^2}\vec{u}_r \cdot \vec{dr}$$

Is $\vec{F}$ conservative?

$$\text{curl } \vec{F} \overset{?}{=} \vec{0} = \vec{C}$$

$$\vec{\nabla} \times \vec{F} = \vec{C} = \vec{0} \,? = \langle C_x, C_y, C_z \rangle$$

$$F_x = -m_1 m_2 G \frac{x}{(x^2+y^2+z^2)^{3/2}}$$

$$F_y = -m_1 m_2 G \frac{y}{(x^2+y^2+z^2)^{3/2}}$$

$$C_z = \partial_x F_y - \partial_y F_x \qquad \partial_x = \frac{\partial}{\partial x}$$

$$-m_1 m_2 G \left[\frac{\partial}{\partial x} \left(\frac{y}{(x^2+y^2+z^2)^{3/2}} \right) - \frac{\partial}{\partial y} \left(\frac{x}{(x^2+y^2+z^2)^{3/2}} \right) \right]$$

$$-m_1 m_2 G \left[-\frac{3}{2} (\quad)^{-5/2} \cdot 2x \cdot y - \left(-\frac{3}{2} (\quad)^{-5/2} 2y \cdot x \right) \right]$$

$$= 0$$

$\vec{F}$ is conservative

How to find U or ΔU?

$$W = -\Delta U$$

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \frac{-m_1 m_2 G}{r^2} \vec{u}_r \cdot d\vec{r}$$

We choose a direct linear path for $\vec{F}$ to follow: $\vec{u}_r \parallel d\vec{r}$

parallel

$$\vec{u}_r \cdot d\vec{r} = 1 \cdot dr$$

$$\int_{r_1}^{r_2} \frac{-m_1 m_2 G}{r^2} dr = \frac{m_1 m_2 G}{r} \Bigg|_{r_1}^{r_2}$$

$$\int \frac{1}{r^2} dr = -\frac{1}{r}$$

$$W = -\Delta U = m_1 m_2 G \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$\Delta U = -m_1 m_2 G \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

We want to define $U(r)$ as a function ~~with its orco~~ with U at the origin set to 0.

r_2 at infinity

$$\boxed{U(r) = -\frac{m_1 m_2 G}{r}} \qquad \vec{F} = -\frac{m_1 m_2 G}{r^2}\vec{u}_r$$

$$-\frac{m_1 m_2 G}{(x^2+y^2+z^2)^{3/2}} \cdot \langle x, y, z \rangle$$

$$U(x,y,z) = -\frac{m_1 m_2 G}{\sqrt{x^2+y^2+z^2}}$$

$$\vec{F} = -\overrightarrow{grad}\, U = -\vec{\nabla} \cdot U(x,y,z)$$

$$= -\left\langle \frac{\partial}{\partial x} U, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right\rangle$$

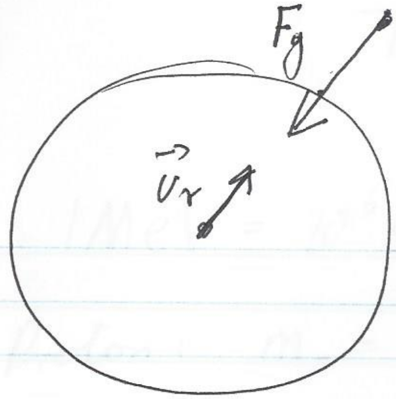
$$F_x = -\frac{\partial U}{\partial x} = m_1 m_2 G \frac{\partial}{\partial x}(x^2+y^2+z^2)^{-\frac{1}{2}}$$

$$= m_1 m_2 G \left(-\frac{1}{2}\right)(\quad)^{-3/2} \cdot 2x \quad ; \text{o.k.}$$

Total energy of a planet in orbit

$$E = \frac{1}{2} m_2 \vec{v}^{\,2} - \frac{m_1 m_2 G}{r} \quad \underline{\text{is constant}}$$

We use ~~Kepl~~ $r = a$ to get a relationship between K and U.



$$\vec{F}_g = m\vec{a}$$

$$\vec{v}_r : -\frac{mMG}{r^2} = -m\frac{v^2}{r}$$

$$= +\frac{m_1 m_2 G}{r^2} = +m_2\frac{v^2}{r}$$

$$\frac{m_1 m_2 G}{r} = m_2 v^2$$

$$E = \frac{1}{2}m_2\vec{v}^2 - \frac{m_1 m_2 G}{r}$$

$$= \frac{1}{2}\frac{m_1 m_2 G}{r} - \frac{m_1 m_2 G}{r}$$

$$= -\frac{1}{2}\frac{m_1 m_2 G}{r} = \frac{1}{2}U$$

Energy of a moving mass is

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

eV = electron Volt

$m_0 c^2$ = rest mass energy

electron: $m_0 = 9.1 \cdot 10^{-31}$ kg

$$E_0 = 9.1 \cdot 10^{-31}\,\text{kg} \cdot \left(3 \cdot 10^8 \frac{m}{s}\right)^2$$

$$= 8.19 \cdot 10^{-14}\,J; \quad \underline{1\,eV = 1.6 \cdot 10^{-19}\,J}$$

$$= 5.12 \cdot 10^5\,eV$$

1MeV = 10^6 eV megaelectronvolt.

Proton: $m_0 = 1.6 \cdot 10^{-27}$ kg

$$m_0 c^2 + \frac{1}{2} m_0 v^2 = E$$

$$m_0 c^2 \gg \frac{1}{2} m_0 v^2$$

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} = m_0 c^2 + K$$

$$\frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} = ?$$

$$(1+x)^n = 1 + nx + \cdots$$

$$X = 0.1 \qquad n = \frac{1}{2}$$

$$1.1^{1/2} = 1.0488$$

$$\frac{m_0 c^2}{\sqrt{}} = m_0 c^2\left(1 + \frac{v^2}{2c^2}\right)$$

$$= m_0 c^2 + \frac{1}{2} m_0 v^2$$

$$1 + nx = 1 + \frac{1}{2} \cdot 0.1 = 1 + 0.05$$

$$\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right)$$

$$= 1 + \frac{v^2}{2c^2}$$

Power = $\frac{\text{work}}{\text{time}}$ in Watts

Instantaneous power: $\frac{dW}{dt} = P(t)$

$$W = \vec{F} \cdot \vec{r}$$

$$\boxed{\frac{dW}{dt} = \vec{F} \cdot \vec{v}}$$

$$1\,\text{Watt} = \frac{1\,\text{Joule}}{1\,\text{sec.}}$$

$$\boxed{1\,kW = 10^3 \frac{J}{s}}$$

Themodynamic energy

radiation energy