$\vec{S}(\vec{r},t)) = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$ Poynting vector, intensity of an electromagnetic field energy per time and area, or power per unit area. Compare this quantity to an energy density current that spreads through space.

(1.1)
$$a)\nabla\times\vec{B} = \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} \text{ and b) } div\vec{B} = 0$$
$$c)\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \text{ and d) div}\vec{E}=0$$

(1.2)
$$u(t) = \frac{\varepsilon_0}{2}\vec{E}\cdot\vec{E} + \frac{1}{2\mu_0}\vec{B}\cdot\vec{B}$$
$$\vec{E} = \vec{E}(\vec{r},t) = \vec{E}(x,y,z,t); \vec{B} = \vec{B}(\vec{r},t) = \vec{B}(x,y,z,t)$$

(1.3)
$$\vec{E}(x,y,z,t) = \vec{E} = \vec{E}_0 e^{i(\vec{k}\bullet\vec{r}-\omega t)}$$
$$\vec{B}(x,y,z,t) = \vec{B} = \vec{B}_0 e^{i(\vec{k}\bullet\vec{r}-\omega t)}$$

(1.4)
$$\frac{\partial}{\partial t}e^{(-i\omega t)} = -i\omega e^{(-i\omega t)}$$
$$div\vec{E} = \vec{\nabla}\cdot\vec{E} \Rightarrow i\vec{k}\cdot\vec{E}$$
$$\vec{\nabla}\times\vec{B} = curl\vec{B} = i\vec{k}\times\vec{B}; \vec{\nabla}\vec{B} = div\vec{B} = i\vec{k}\vec{B}$$
$$\vec{\nabla}U = \overrightarrow{grad}U = i\vec{k}U$$
$$\vec{\nabla}\bullet\vec{\nabla} = \Delta = Laplace = (i\vec{k})^2 = -k^2$$

(1.5)
$$\bar{S}_{avg} = \frac{1}{2}\frac{E_{\max}^2}{\mu_0 c} = \frac{1}{2}\frac{cB_{\max}^2}{\mu_0}$$

(1.6)
$$div\vec{S} = -\frac{\partial u}{\partial t}$$

(1.7)
$$\frac{S}{c} = u \quad S = uc$$

(1.8)
$$\frac{S}{c} = u = \frac{U}{V} = \frac{Fdx}{Adx} = pressure\ P$$

(1.9)
$$\frac{S}{c} = u = \frac{U}{V} = \frac{F\,\cancel{dx}}{A\,\cancel{dx}} = \frac{dp}{Adt}$$
$$dp = \frac{SAdt}{c} = \frac{SAdxdt}{cdx} = \frac{SV}{c^2} = \frac{uV}{c} = \frac{U}{c}$$

(1.10) $$\boxed{dp = \frac{U}{c}}$$

Maxwell's Equations and Hertz's discoveries.

1. Calculate, expand: $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) = \varepsilon_0 \mu_0 \vec{\nabla} \times \frac{\partial \vec{E}}{\partial t}$
   and show that the result is the wave equation for the magnetic field.
2. Calculate the following expressions with $\vec{k} = \langle 2,3,4 \rangle / m;\ \vec{r} = \langle x, y, z \rangle$
   1. $\frac{\partial}{\partial x}\left(B_0 \sin\left(\vec{k}\cdot\vec{r} - \omega t\right)\right); \frac{\partial}{\partial t}\left(B_0 \sin\left(\vec{k}\cdot\vec{r} - \omega t\right)\right)$
   2. $\frac{\partial}{\partial x}\left(B_0 e^{i\left(\vec{k}\bullet\vec{r} - \omega t\right)}\right)$ *and* $\frac{\partial}{\partial t}\left(B_0 e^{i\left(\vec{k}\bullet\vec{r} - \omega t\right)}\right)$
   3. With $\vec{E}\left(\vec{r}, t\right) = \langle E_x, E_y, E_z \rangle e^{i\left(\vec{k}\bullet\vec{r} - \omega t\right)}$ show by explicit calculation that:
      $div\vec{E} = i\vec{k}\bullet\vec{E},\ curl\vec{E} = i\vec{k} \times \vec{E},\ \frac{\partial}{\partial t}\vec{E} = -i\omega\vec{E}$
3. A very long thin rod carries electric charge with the linear density 35.0 nC/m. It lies along the x-axis and moves in the x-direction at a speed of 15.0E6m/s. a)Find the electric field the rod creates at the point (0, 20cm, 0). (Gauss's law.) b) Find the magnetic field it creates at the same point. (Ampere's law).
   c)Find the force exerted on an electron moving with a velocity of $240E6\frac{m}{s}\vec{\mathrm{i}}$
   a)3.15kV/m $\vec{\mathrm{j}}$ b)5.25E-7T $\vec{\mathrm{k}}$ c)-4.84E-16N $\vec{\mathrm{j}}$
4. (no)Prove by explicit calculation that: $div\left(\vec{E} \times \vec{B}\right) = \vec{B}\left(\vec{\nabla} \times \vec{E}\right) - \vec{E}\left(\vec{\nabla} \times \vec{B}\right)$ Use the definition of div and the definition of the cross-product.
5. Which of the following expressions is defined, and why, or why not:
   $a)\ \overrightarrow{grad}\left(\vec{A} \times \vec{B}\right); b)\ \overrightarrow{grad}\left(\vec{A} \cdot \vec{B}\right)$
   $c) div\left(\vec{A} \times \vec{B}\right); d) div\left(\vec{A} \cdot \vec{B}\right)$
   $e) curl\left(\vec{A} \times \vec{B}\right); f) curl\left(\vec{A} \cdot \vec{B}\right)$

Plane electromagnetic waves.

6. Draw the picture of a plane em wave propagating in the z-direction. Suppose that the wavelength is 50.0m, and the electric field vibrates in the x-z plane with an amplitude of 22.0V/m. a) calculate the frequency of the wave and b)

the magnitude and direction of $\vec{B}$ when the electric field has its maximum value in the negative x direction c) write an expression for $\vec{B}$ with the correct unit vector with numerical values for $B_{max}$, the wave number, and angular frequency, and as a sine function, exponential function.

d) Find the magnitude of the Poynting vector and draw it in your picture.

e) Find the average energy of this wave contained in 10cm$^3$ of space.

a) f=6.00MHz; b) $\left|\vec{B}_{\max}\right| = -73.3\vec{j}\text{nT}$ c) $\vec{B} = -73.3\sin(0.126z - 3.77E7t)\left(-\vec{j}\right)$ or $\vec{B} = -73.3e^{i(0.126x-3.77E7t)}\text{nT}\cdot\left(-\vec{j}\right)$

d) S=1.28W/m$^2$, z-direction. e) 2.13E-14J

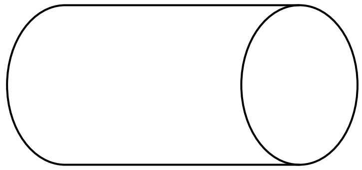
7. In SI units, the electric field of an em wave is described by $E_y = 100\frac{V}{m}\sin\left(1.00E7x - \omega t\right)$. Find a) the amplitude of the corresponding magnetic field oscillations and its direction, b) the wavelength and wave-number c) the frequency f and angular frequency. d) Calculate the maximum value of the Poynting vector.

a) B=333nT ; z-direction b) $\lambda = 628nm; k = 10^7 m^{-1}$; c) f=4.77E14Hz; =3.00E15 s$^{-1}$ d) S=26.5W/m$^2$.

8. (11) How much electromagnetic energy per cubic meter is contained in sunlight, if the intensity of sunlight at the Earth's surface under a fairly clear sky is 1kW/m$^2$?

$$S = \text{intensity}=\frac{\text{power}}{\text{A}} = \frac{U/t}{A} = \frac{U/t}{V/x} = \frac{U}{V}c \Rightarrow \frac{S}{c} = u = 3.33\frac{\mu J}{m^3}$$

9. (13) What is the average magnitude of the Poynting vector 5.00 miles from a radio transmitter broadcasting isotropically with an average power of 250kW? (3.07E-4 W/m$^2$).
10. (17) The filament of an incandescent lamp has a 150 Ohm resistance and carries a direct current of 1.00A. The filament is 8.00cm long and 0.900mm in radius. Draw the vectors for the electric, magnetic field, and the Poynting vector. (a) calculate the Poynting vector at the surface of the filament associated with the static electric field producing the current and the current's static magnetic field. b) Find the magnitude of the static electric and magnetic fields at the surface of the filament. Hints: Power =RI$^2$; S=power/$A_{cylinder}$; S points radially inward. B comes from the current. E comes from $\frac{\Delta V}{\Delta s}$; ΔV=RI. ( $S = 331kW/m^2; E = 1.88kV; B = 2.22E-4\,T$ )



11. (19) In a region of free space the electric field at an instant of time is $\vec{E} = \langle 80.0, 32.0, -64.0 \rangle V/m$ and the magnetic field is $\vec{B} = \langle 0.200, 0.080, 0.290 \rangle \mu T$ a) Show that the two fields are perpendicular to each other, b) determine the Poynting vector for these fields.

Momentum and radiation pressure

12. (27) A radio wave transmits 25.0 W/m$^2$. A flat surface of area A is perpendicular to the direction of propagation of the wave. Calculate the radiation pressure on it, assuming the surface is a perfect absorber.

(P=U/c =8.33E-8N/m$^2$)

13. (29) A 15.0 mW helium-neon laser ( $\lambda = 632,8nm$ ) emits a beam of circular cross section with a diameter of 2.00 mm. a) Find the maximum electric field in the beam. b) What total (average) energy is contained in a 1.00 m length of the beam? c) Find the momentum carried by a 1.00m length of the beam.

(S=power/A=4.78kW/m$^2$, $\bar{S} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E^2_{\max}}{2c\mu_0}$ ;$E_{max}$=1.90kV/m; U=5E-11J=50.0pJ, p=U/c=1.67E-19 kg·m/s)

14. Show that the argument of a linearly polarized e.m. wave can be written as follows: $kx - \omega t = \frac{2\pi}{\lambda}(x - ct)$