(1.1) $\Delta V(t) = R \cdot I(t)$
$\Delta V_{\max} e^{i\omega t} = R I_{\max} e^{i\omega t}$

(1.2) $P(t) = I \cdot \Delta V = R \cdot I^2 = I_{\max} \cdot \Delta V_{\max} \cdot \sin^2 \omega t$

$$\frac{1}{T}\int_0^T \sin^2\omega t \cdot dt = \frac{1}{2}; \cos^2\theta = \frac{1}{2}(1+\cos 2\theta); \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

(1.3) $P_{av} = I_{\max}\Delta V_{\max}\frac{1}{2} = \frac{I_{\max}}{\sqrt{2}}\frac{\Delta V_{\max}}{\sqrt{2}} = I_{rms} \cdot \Delta V_{rms}$

(1.4) $\Delta V_{rms} = \frac{\Delta V_{\max}}{\sqrt{2}} = 0.707\Delta V_{\max}$
$I_{rms} = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max}$

(1.5) $\Delta\hat{V} = \Delta V_{\max} e^{i\omega t} = \hat{z}_{eq}\hat{I} = |\hat{z}_{eq}| I_{\max} e^{i(\omega t+\phi)}$
$|\hat{z}_{eq}| = \sqrt{X^2+Y^2}$; X is the real part of $\hat{z}_{eq}$ and
Y is the imaginary part of $\hat{z}_{eq}$. $\phi = \tan^{-1}\left(\frac{Y}{X}\right)$

(1.6) $\hat{I}(t) = \frac{\hat{V}(t)}{\hat{Z}}$;
$\hat{Z}_L = i\omega L \Rightarrow \hat{I}(t) = \frac{\hat{V}(t)}{i\omega L}; I_{\max} = \frac{V_{\max}}{\omega L}$
$\hat{Z}_C = \frac{1}{i\omega C} \Rightarrow \hat{I}(t) = i\omega C\hat{V}(t); I_{\max} = \omega C V_{\max}$

Resistors in an ac circuit.

1. The rms output voltage of an ac source is 200V and the operating frequency is 100Hz. Write the equation giving the output voltage as a function of time.
   V(t)=$\sqrt{2}\cdot 200V\cos(2\pi\cdot 100t)$ or $\sin\omega t$ or $e^{i\omega t}$
2. Write the complex number $\hat{z} = 3+4i$ in exponential form.
   Find the magnitude and the phase of the number. (5; θ=0.0.927)
3. Write the complex expression $(3+4i)e^{i\omega t}$ in terms of a real amplitude times an exponential expression including the phase of the complex amplitude. ($5e^{i(\omega t+0.927)}$)
4. Draw the following complex voltages $\hat{V}_1 = 3ie^{i\omega t}; \hat{V}_2 = -5ie^{i\omega t}; \hat{V}_3 = 5e^{i\omega t}$ on a Gaussian complex plane and add them together, both graphically and mathematically. What is the resultant of the sum in algebraic and exponential,

and trigonometric form, the magnitude, and the phase shift?

$$(5-2i)e^{i\omega t}=\sqrt{29}e^{i(\omega t-0.381)}=5.39e^{i(\omega t-0.381)}=$$
$$5.39\left(\cos(\omega t-0.381)+i\sin(\omega t-0.381)\right)$$

5. The voltage across a capacitor is given by
$$\Delta\hat{V}_C=\frac{1}{i\omega C}\hat{I};\ \hat{I}=6.00Ve^{i(\omega t+0.23)};\omega=377s^{-1}\ \mathrm{C=25\mu F}$$
Find the maximum voltage and the new phase shift with respect to the current.
(637V, -1.34)
6. (3) An ac power supply produces a maximum voltage $\Delta V_{\max}=100V$. This power supply is connected to a 24.0 Ohm resistor, and the current and resistor voltage are measured with an ideal ac ammeter and voltmeter. What does each meter read?
$$\Delta V=\frac{\Delta V_{\max}}{\sqrt{2}}=0.707\cdot 100V=70.7V;I=\frac{70.7V}{24.0\Omega}=2.95A$$

Capacitors in an ac-circuit:

7. You are dealing with the following RC-circuit:
$$R=150\Omega;\ \omega=150/s;\varepsilon_{\max}=310V;C=25\mu F$$

   a) Find the complex impedance and complex current of the circuit. Write your results in terms of the magnitude and the exponential value.
   $$\hat{Z}_{eq}=306\Omega e^{-i1.06};\Phi=-1.06;\hat{I}=1.01Ae^{i(\omega t+0.1.06)}$$

   b) Find the maximum voltage across the capacitor.
   $270V$
   c)

Inductors in an ac circuit:

8. (9) In a purely inductive ac circuit $\Delta V_{\max}=100V$. The maximum current is 7.50A at 50Hz. Calculate the inductance L.
L=42.5mH
At what angular frequency is the maximum current 2.50A? (942/s)
9. (11) For a purely inductive circuit with source voltage $\varepsilon_{\max}\sin\omega t$ we are given $\varepsilon_{\max}$=80.0 V, f=32.5Hz, L=70.0mH. Calculate the instantaneous current in the inductor at 15.5ms. (I_{max}= 5.6A,
$$\sin\left(\omega t-\frac{\pi}{2}\right)=\sin 0.7175=0.657$$)
10. (13) Determine the maximum magnetic flux through an inductor connected to a standard electrical outlet, $\Delta V_{rms}=120V,\ f=60.0Hz$ Hint: $L=\frac{\Phi_B}{I}$
(0.45Wb)

11. In an LR circuit: R=50.0Ω, L=0.900H in series; applied maximum voltage=100V, ω=200/s. Find the complex impedance, the complex current , its maximum and phase shift. Find the average power-loss.
    $\hat{Z}_{eq} = 187e^{i1.30}\Omega; \hat{I} = 0.535e^{i(\omega t - 1.30)}A; \bar{P} = 7.15W$

Capacitors in an AC circuit:

12. (15) What is the maximum current in a 2.20E-6F capacitor when it is connected across a) a North American electrical outlet with 120V rms and 60Hz, and b) a European outlet with 240V rms and 50.0Hz?

(a) 0.141A, b) 0.234A

The RLC circuit:

13. (19) An inductor (400mH), a capacitor (4.43E-6F) and a resistor (500Ohm) are connected in series. A 50.0Hz source produces a peak current of 250 mA in the circuit. Calculate the required peak voltage and the phase angle by which the current leads or lags the applied voltage. (V_{max}=194V, Z=776 Ohm, -49.9°=-0.87radians)

Power in an AC circuit

14. (31)An AC voltage of the form 100Vsin(1000t) is applied to a series RLC circuit. The resistance is 400 Ohm, the capacitance is 5.00E-6 F, and the inductance is 0.500H. Find the average power delivered to the circuit. (8.00W)
15. no(35) Suppose power P is to be transmitted over a distance L at a voltage ΔV with only 1.00% loss. Copper wire of what diameter should be used for each of the two conductors of the transmission line? Assume that the current density in the conductors is uniform. Express the diameter in terms of P, ΔV, L, and ρ. (Hint: In order to complete a loop you need two wires.P=ΔV·I, loss=P/100=RI^2; R=ρ·4·2L/πd^2)

($2r = \sqrt{\dfrac{800\rho PL}{\pi(\Delta V)^2}}$)

Resonance in Serial RLC circuits

16. (37) An RLC circuit is used in radio to tune into an FM station broadcasting at 99.7MHz. The resistance in the circuit is 12.0 Ohm, and the inductance is 1.40E-6H. What capacitance should be used? (1.82pF)
17. (41) A 10.0 Ohm resistor, 10.0 mH inductor, and 0.1mF capacitor are connected in series to a 50.0V rms source having variable frequency. Find the energy that is delivered to the circuit during one period if the operating frequency is twice the resonance frequency. (242mJ)
18. A resistor of 150 Ohms, and an inductor of 0.800H, are connected in parallel to an ac source of 150V, and 60Hz. Find the equivalent impedance and the current in the equivalent circuit. Find the maximum currents in the resistor branch and in the inductor branch.

$$\hat{Z}_{eq} = \frac{i\omega LR}{R+i\omega L} = \frac{i\omega LR}{R+i\omega L}\frac{R-i\omega L}{R-i\omega L} = \frac{R(\omega L)^2 + i\omega LR^2}{R^2+(\omega L)^2} = 121 + i59.8$$

$$\left|\hat{Z}_{eq}\right| = 134\Omega; \Phi = 26.3° = 0.459$$

$$\hat{I} = 1.12Ae^{i(\omega t - 0.459)}; I_R = 1.00A; I_L = 0.498A$$

Do the currents add up to the main current?

Transformers and power transmission.

19. (45) A transformer has N_1=350 turns and N_2=2000 turns. If the input voltage is $170V\cos\omega t$ what rms voltage is developed across the secondary coil? (687V rms)

20. (49) A transmission line that has a resistance per unit length of $4.50\cdot 10^{-4}\frac{\Omega}{m}$ is to be used to transmit 5.00MW over 400 miles. The output voltage of the generator is 4.50kV.

a) What is the line loss if a transformer is used to step up the voltage to 500kV?
b) What fraction of the input power is lost to the line under these circumstances?
c) What difficulties would be encountered in attempting to transmit the 5MW at the generator voltage of 4.50kV?
(a) 29kW; R=288Ω b)0.58% c) melt down.