33 Circuits With Alternating Voltage Sources. See website homework.pdf

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33.1 Introduction: Choice of functions for the ac source.

33.1a Summary:

In this chapter we are going to study circuits with capacitors, resistors, and coils driven by an alternating power source of the form $V(t)=V_{\max}\sin\omega t$. We need to clearly distinguish between time dependent parts of such functions and their time-independent amplitudes.

I will stick to the topics of chapter 33 in the textbook by Serway, but use complex numbers and functions in order to derive the results. We use

(33.1) $$\hat{I}(t)=\hat{I}_{\max}e^{i\omega t} \text{ and } \hat{V}(t)=\hat{V}_{\max}e^{i\omega t}$$

current : I; *complex current* $\hat{I}(t)=\hat{I}_{\max}e^{i\omega t};\hat{I}_{\max}$ = complex amplitude; *i imaginary unit*

Why do I do this? Just consider how you would calculate the ac-current in a loop containing a resistor, a capacitor and a coil in series.

(33.2) $$V(t)=RI(t)+\frac{Q(t)}{C}+L\frac{dI(t)}{dt}$$

The derivative of a sine function gives you a cosine function, and the antiderivative gives you a negative cosine function. You end up with a sum of functions which you cannot factor out. Now look what happens when you use complex functions.

(33.3) $$\hat{I}(t)=\hat{I}e^{i\omega t} \text{ and } \hat{V}(t)=\hat{V}e^{i\omega t}$$
$$\frac{\hat{Q}(t)}{C}=\frac{1}{C}\int \hat{I}e^{i\omega t}dt=\frac{1}{i\omega C}\hat{I}e^{i\omega t} \text{ and } L\frac{d\hat{I}}{dt}=i\omega L\hat{I}e^{i\omega t}$$

The charge Q(t) is the antiderivative of I(t), while the impedance involves the derivative of I(t). If you use these complex functions the antiderivative corresponds to a division by iω, and the derivative corresponds to a multiplication by iω. Equation (33.2) becomes an algebraic relationship between complex quantities:

(33.4) $$\hat{V}_{\max}e^{i\omega t}=R\hat{I}(t)+\frac{1}{i\omega C}\cdot\hat{I}(t)+i\omega L\cdot\hat{I}(t)=\left(R+\frac{1}{i\omega C}+i\omega L\right)\hat{I}_{\max}e^{i\omega t}$$

This means we have managed to write the relationship between current and voltage in an ac circuit in the same way as in a dc circuit.

(33.5) $$V=R_{eq}I\Rightarrow\hat{V}(t)=\hat{Z}_{eq}\cdot\hat{I}(t);\ \hat{Z}_{eq} \text{ is called the equivalent impedance.}$$

Solving for the current leads to:

(33.6) $$\hat{I}(t)=\frac{1}{\hat{Z}_{eq}}\hat{V}_{\max}e^{i\omega t}$$

If we write the equivalent impedance in terms of a magnitude multiplied by an exponential phase Φ, we get:

(33.7) $$\hat{Z}_{eq}=a+ib=|a+ib|e^{i\Phi}=\sqrt{a^2+b^2}e^{i\Phi};\ \Phi=\tan^{-1}\frac{b}{a}$$
$$\hat{Z}_{eq}=\left|\hat{Z}_{eq}\right|e^{i\Phi}$$

Inserting this into (33.6) gives us:

(33.8) $$\hat{I}(t)=\left|\frac{\hat{V}_{\max}}{\hat{Z}_{eq}}\right|e^{i(\omega t-\Phi)}$$

Thus, the magnitude of the current is equal to V_{max}/Z_{eq}. The argument of the sine, or cosine function of the current is shifted by -Φ; $\Phi = \tan^{-1}\frac{b}{a}$.

The most obvious difference between a dc voltage and an ac voltage, is that ac voltage changes polarity. It is negative and positive half the time. This holds of course for the current as well. In the case of a dc circuit we found that the relationship between voltage and current could always be written in terms of $V = R_{eq}I$ All we had to do was apply Kirchhoff's law and find the equivalent resistance R_{eq}. Solving for I, we could then find the current in the circuit. Reminder of Kirchhoff's loop law:

(33.9) The algebraic sum of all voltages around any closed loop in a circuit must be 0 at any instant in time. $\sum_k \Delta\hat{V}_k = 0$

A simple resistor in a circuit will dissipate the power supplied by the exterior voltage, with the current in the circuit directly proportional to the voltage, at all times, meaning that the voltage and the current across a resistor are in phase. If the voltage is a sine function, so is the current. We apply Kirchhoff's law to the circuit:

(33.10) Kirchhoff's rule say that we have to count the voltage drop negatively if we pass through the resistor following the direction of the current, because in that case we go from a higher potential to a lower potential.

$$V(t) + \Delta V_R = 0; V(t) - RI(t) = 0$$

(33.11)

$$V(t) \equiv \varepsilon(t) = \varepsilon_{\max}\sin\omega t = R\cdot I(t) = RI_{\max}\sin\omega t$$

$$\text{In complex notation:}\ \hat{\varepsilon}(t) = \varepsilon_{\max}e^{i\omega t} = R\hat{I}(t) = RI_{\max}e^{i\omega t}$$

There is no phase shift between the current and the voltage for the voltage/current relationship. Both voltage and current have the identical sine function (or cosine function.)

We want to find a similarly simple way of expressing the relationship between ac current $\hat{I}(t)$ and the applied ac voltage $\hat{V}(t) \equiv \hat{\varepsilon}(t)$ in circuits, containing so-called **passive elements** like resistors, capacitors (condensors), and coils, with their respective resistance, capacitance, and inductance.

(33.12)

$$\boxed{\hat{V}(t) = \hat{Z}_{eq}\cdot\hat{I}(t)}$$

$\hat{Z}_{eq}$ is a complex number, which we will write in terms of its magnitude and its exponential phase.

(33.13)

$$\hat{Z}_{eq} = a + ib = \sqrt{a^2+b^2}\cdot e^{i\Phi}; \Phi = \tan^{-1}\frac{b}{a}$$

The real part of the power-source is an alternating voltage for which we use the notation:

(33.14)

$$\varepsilon(t) = V(t) = V_{\max}\sin\omega t$$

We could also use a cosine function for the applied voltage, as some textbooks do, but instead we are going to make use of complex numbers here and use for the applied voltage or emf:

(33.15) $\hat{\varepsilon}(t) = \hat{V}(t) = V_{\max} \cdot \underbrace{e^{i\omega t}}_{\text{complex time-dependency}} \; ; V_{\max} (\text{or } \varepsilon_{\max}) = \text{magnitude of the applied voltage}$

For the current we use the same complex notation:

(33.16) $$\boxed{\hat{I}(t) = \underbrace{\hat{I}_{\max}}_{\text{complex amplitude}} e^{i\omega t} = I_{\max} e^{-i\Phi} e^{i\omega t} = I_{\max} e^{i(\omega t - \Phi)}}$$

The phase Φ comes from the equivalent impedance $\hat{Z}_{eq}$ which is the sum of all the impedances in a circuit branch for which we calculate the current. As noted previously, in the case of a resistor, a coil, and a capacitor in series, this is equal to:

(33.17) $$\hat{Z}_{eq} = R + i\omega L + \frac{1}{i\omega C} = R + i\left(\omega L - \frac{1}{\omega C}\right) = a + ib = \sqrt{a^2 + b^2} e^{i\Phi}; \; with\ \Phi = \tan^{-1}\frac{b}{a}$$

$$\hat{V}(t) e^{i\omega t} = V_{\max} e^{i\omega t} = \hat{Z}_{eq} \hat{I}(t) = \left|\hat{Z}_{eq}\right| e^{i\Phi} \hat{I}_{\max} e^{i\omega t} \Rightarrow$$

(33.18) $$\hat{I}(t) = \frac{V_{\max}}{\left|\hat{Z}_{eq}\right|} e^{i(\omega t - \Phi)}$$

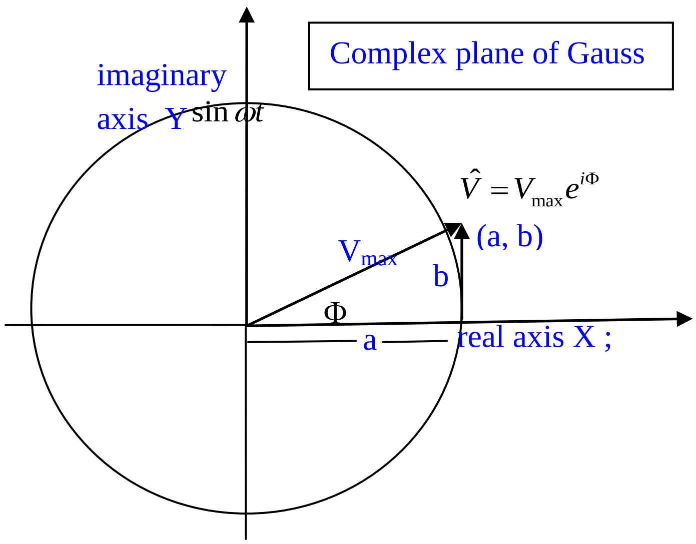
The advantages of the complex notation will become apparent as we go along. **The relationship between the complex amplitudes of voltage and currents** can be described graphically through **phasors** or through their relationship in the complex plane of Gauss.
It ultimately does not matter whether we use a cosine function or sine function for the applied voltage. The power in a circuit is the product between voltage and current. Their average value depends only on the maximum value for current and voltage and on the phase shift between them. (See later.) The two **new** passive elements in a ac circuit are the inductance L and the capacitance C. They produce a phase shift in the sinosoidal function of $\pm\frac{\pi}{2}$.

33.2 Review of Complex Numbers, (see file "ch32 Complex Oscillations").
Carl Friedrich Gauss introduced complex numbers: In the complex plane the y-axis is the imaginary axis and the x-axis is the real axis. Any complex number $\hat{z}$ can then be written in terms of its real part x plus its imaginary part y. I usually use the the hat $\hat{z}$ on top of the complex number, or function. Note, that when we talk about the imaginary part y, we do not include the imaginary unit i.

$$\hat{z} = x + iy \Rightarrow$$

(33.19) $$\hat{z} = r(\cos\theta + i\sin\theta) = re^{i\theta} \text{ Euler formula}$$

One can easily prove Euler's formula by expanding the exponential function e^x in a McLaurin power series, and then substituting x with iθ. We must just use that $i^2 = -1$; $i^3 = -i$; $i^4 = 1$; $i^5 = i$



Note that Φ is the angle between the horizontal axis and the magnitude of the complex number. Note in particular that:

(33.20) $$i = e^{i\frac{\pi}{2}} \; and \; \frac{1}{i} = -i = e^{-i\frac{\pi}{2}}$$

$$e^{i\pi} = -1$$

If you add π/2 (or any other angle to any vector you rotate the vector ccw by this angle.) If you subtract it, you rotate the vector in the clockwise direction. This is convenient if you want to determine the relationships between the following functions:

(33.21)
$$\cos\left(\omega t + \frac{\pi}{2}\right) = -\sin\omega t; \cos\left(\omega t - \frac{\pi}{2}\right) = \sin\omega t$$
$$\sin\left(\omega t + \frac{\pi}{2}\right) = \cos\omega t; \sin\left(\omega t - \frac{\pi}{2}\right) = -\cos\omega t$$

Taking the **derivative** of an exponential complex function corresponds to rotating its vector by π/2 **counter clockwise**:

$$\frac{d}{d\theta}e^{i\theta} = ie^{i\theta} = e^{i\frac{\pi}{2}+i\theta} = e^{i\left(\theta+\frac{\pi}{2}\right)} = \cos\left(\theta+\frac{\pi}{2}\right) + i\sin\left(\theta+\frac{\pi}{2}\right) =$$

(33.22) $$\frac{d}{d\theta}(\cos\theta + i\sin\theta) = -\sin\theta + i\cos\theta \Rightarrow \cos\left(\theta+\frac{\pi}{2}\right) = -\sin\theta \; and \; \sin\left(\theta+\frac{\pi}{2}\right) = \cos\theta$$

We are equating the real and the imaginary parts of the complex function with each other.

Taking the **anti-derivative** of an exponential complex function corresponds to rotating its vector by π/2 **clockwise**:

(33.23)
$$\int e^{i\theta}d\theta = \frac{e^{i\theta}}{i} = e^{-i\frac{\pi}{2}+i\theta} = e^{i\left(\theta-\frac{\pi}{2}\right)} = \cos\left(\theta-\frac{\pi}{2}\right) + i\sin\left(\theta-\frac{\pi}{2}\right) =$$
$$\int(\cos\theta + i\sin\theta)d\theta = \sin\theta - i\cos\theta \Rightarrow \cos\left(\theta-\frac{\pi}{2}\right) = \sin\theta \; and \; \sin\left(\theta-\frac{\pi}{2}\right) = -\cos\theta$$

33.3 Capacitors in AC-circuits:

For example, inside of an ac circuit with a capacitor, the voltage drop across the capacitor is

(33.24) $$\Delta V_C(t) = \frac{Q(t)}{C}$$

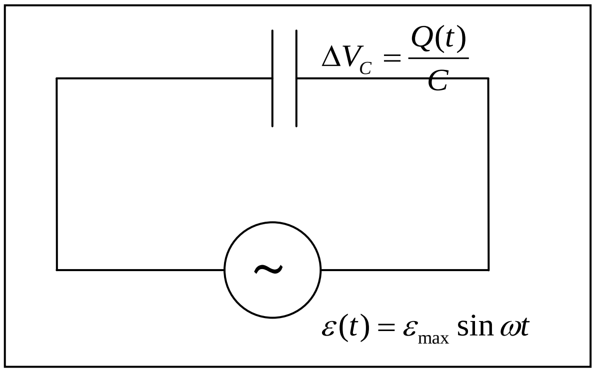
With a steady dc current, once the capacitor is charged, the circuit behaves like an open circuit, no current is flowing any more.

With an ac current the **voltage difference across the capacitor oscillates back and forth with the charge**, because the charge is proportional to the applied voltage. We apply Kirchhoff's rule, noting that we traverse the capacitor from a higher to a lower potential:

(33.25) $$\varepsilon(t)+\Delta V_C=0 \Rightarrow \varepsilon(t)-\frac{Q(t)}{C}=0$$

(33.26) $$\varepsilon_{\max}\sin\omega t=\frac{Q(t)}{C} \Rightarrow Q(t)=C\cdot\varepsilon_{\max}\sin\omega t$$

But what about the current? We get the current by taking the derivative of the charge:



(33.27) $$I(t)=\frac{dQ(t)}{dt}=\omega C\varepsilon_{\max}\cos\omega t$$
$$I(t)=\omega C\varepsilon_{\max}\sin\left(\omega t+\frac{\pi}{2}\right)$$

This shows that the current in a circuit with a capacitor leads the applied voltage by π/2. What matters physically is this phase relationship between the current in a circuit and the applied voltage.

33.3a Using Complex Functions To Determine The Current:

We determine the current across a capacitor by first finding the complex charge and then taking the derivative. The exponential forms of our functions are particularly easy to deal with:

(33.28) $$\frac{d}{dt}e^{i\omega t}=i\omega e^{i\omega t} \Leftrightarrow \text{derivative = multiplication with } i\omega$$
$$\int e^{i\omega t}dt=\frac{1}{i\omega}e^{i\omega t} \Leftrightarrow \text{ anti-derivative = division by } i\omega$$

Applying Kirchhoff's law to the loop, we traverse the capacitor in the direction of the positive current from a higher to a lower potential, hence:

(33.29) $$\hat{\varepsilon}(t)+\Delta\hat{V}_C=0 \Rightarrow \hat{\varepsilon}(t)-\frac{\hat{Q}(t)}{C}=0$$

(33.30) $$\hat{Q}_C(t)=C\underbrace{\varepsilon_{\max}e^{i\omega t}}_{\substack{applied\\ voltage}}$$

We obtain the current in the loop by taking the time derivative of the charge :

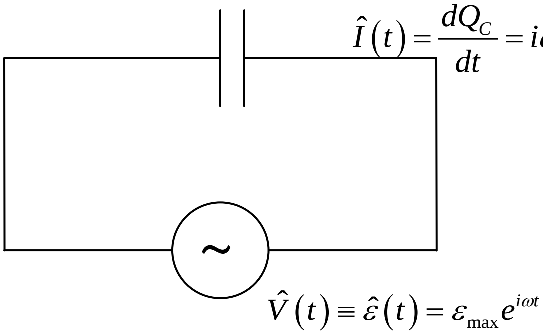
(33.31) $$\hat{I}(t)=\frac{d\hat{Q}}{dt}=i\omega C\underbrace{\varepsilon_{\max}e^{i\omega t}}_{\hat{V}(t)}=i\omega C\hat{\varepsilon}(t)$$

Note that we made use of the fact that the imaginary unit "i" is equal to $e^{i\frac{\pi}{2}}$.

$$\hat{I}(t) = \omega C \varepsilon_{\max} e^{i\left(\omega t + \frac{\pi}{2}\right)} = \text{oscillating current of the capacitor}$$

The current leads the applied voltage by $\frac{\pi}{2}$.

If we choose a sine function for the applied voltage we get the result in (33.27)



The current **leads** the voltage by $\frac{\pi}{2}$, which amounts to the same as saying that the voltage **lags** behind the current by $\frac{\pi}{2}$.

The voltage across a capacitor in terms of the current is given by:

(33.32) $\Delta\hat{V}_C(t) = \frac{\hat{I}(t)}{i\omega C} = \frac{I_{\max}}{i\omega C} e^{i\omega t} = \frac{I_{\max}}{\omega C} e^{i\left(\omega t - \frac{\pi}{2}\right)}$

In our approach we use the **complex capacitive impedance** $\hat{Z}_C = \frac{1}{i\omega C}$ and write down Kirchhoff's law for the purely capacitive circuit, like this

(33.33) $$\boxed{\hat{V}_C(t) = \frac{1}{i\omega C}\hat{I}(t) = \hat{Z}_C \hat{I}(t)}$$

We write the complex impedance in terms of a magnitude and an exponential phase:

(33.34) $$\hat{Z}_C = \frac{1}{i\omega C} = \left|\hat{Z}_{eq}\right| e^{i\Phi} = \frac{1}{\omega C} e^{-i\frac{\pi}{2}} \text{ because } \frac{1}{i} = -i = e^{-i\frac{\pi}{2}}$$

We now multiply the complex portion of the impedance into the complex exponential portion of the current :

(33.35) $$\Delta\hat{V}_C = \hat{Z}_C \hat{I}(t) = \frac{1}{\omega C} e^{-\frac{\pi}{2}i} I_{\max} e^{i(\omega t)} = \frac{I_{\max}}{\omega C} e^{i\left(\omega t - \frac{\pi}{2}\right)}$$

In a purely capacitive circuit, this represents the voltage drop across the capacitor.
Recall that our goal is to find formulas for the voltage drop across a capacitor or an inductor, which are similar to Ohm's law in the form of V=RI. **We call these coefficient between the voltage and the current in an ac circuit: impedance.**

Memorize: the complex impedance for a capacitor is:

(33.36) $$\boxed{\hat{Z}_C = \frac{1}{i\omega C}}$$

This looks harder than it really is.

Let's assume the following values: $\omega = 500/s; \varepsilon_{\max} = 150V; C = 25\mu F$

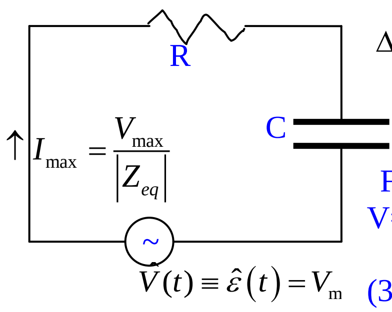
We calculate $Z = \frac{1}{\omega C} = 80\Omega; \Phi = -\frac{\pi}{2}; I_{\max} = \frac{V_{\max}}{Z} = \frac{150V}{80\Omega} = 1.87A; \hat{I}(t) = 1.87Ae^{i\left(500t+\frac{\pi}{2}\right)}$

Some texts use the term **capacitive reactance** for for the magnitude Z_C. (Both names, reactance and impedance indicate the fact that an inductor or a capacitor **impede** the current, or **act** against it.)

33.4 RC Circuits (R and C in series):

Let us now introduce a resistor into the circuit to get an RC circuit, with an ac power supply.

Kirchhoff's law: $\hat{\varepsilon}(t) + \Delta\hat{V}_C + \Delta\hat{V}_R = 0 \Rightarrow \hat{\varepsilon}(t) - \frac{\hat{Q}(t)}{C} - R\hat{I}(t) = 0$ We just found the voltage drop across the capacitor in terms of the complex current $\Delta\hat{V}_C = \frac{1}{i\omega C}\hat{I}$. If we use that in our equation we get:



(33.37) $$\hat{\varepsilon}(t) - \frac{1}{i\omega C}\hat{I}(t) - R\hat{I}(t) = 0$$

Factoring out the current we get a relationship reminiscent of V=RI for dc circuits, namely:

(33.38) $$\boxed{\hat{\varepsilon}(t) = \underbrace{\left(R + \frac{1}{i\omega C}\right)}_{\hat{Z}_{eq}=\text{equivalent impedance}}\hat{I}(t) = 0}$$

The equivalent impedance is a always a complex number which we write as a complex exponential number, using Euler's formula:

(33.39) $$\boxed{\hat{Z}_{eq} = a + ib = \left|\hat{Z}_{eq}\right| e^{i\Phi} = \sqrt{a^2 + b^2}\, e^{i\Phi};\ \text{with } \Phi = \tan^{-1}\left(\frac{b}{a}\right)}$$

Written in this form, it is particularly easy to determine the maximum current and the phase shift between the current and the applied voltage:

(33.40)

$$\hat{\varepsilon}(t) = \hat{Z}_{eq}\hat{I} = \left(R + \frac{1}{i\omega C}\right)\hat{I} = \left(R - \frac{i}{\omega C}\right)\hat{I} = \left|\hat{Z}_{eq}\right| e^{i\Phi}\hat{I} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}\, e^{i\Phi}\hat{I}; \Phi = \tan^{-1}\left(\frac{-1/\omega C}{R}\right)$$

We now can easily get the complex current, its maximum value, and the phase shift:

(33.41) $$\hat{I}(t)=\frac{\hat{\varepsilon}(t)}{\hat{Z}_{eq}}=\frac{\varepsilon_{\max}e^{i\omega t}}{\left|\hat{Z}_{eq}\right|e^{i\Phi}}=\frac{\varepsilon_{\max}}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}e^{i(\omega t-\Phi)};\Phi=\tan^{-1}\left(\frac{-1/\omega C}{R}\right)$$

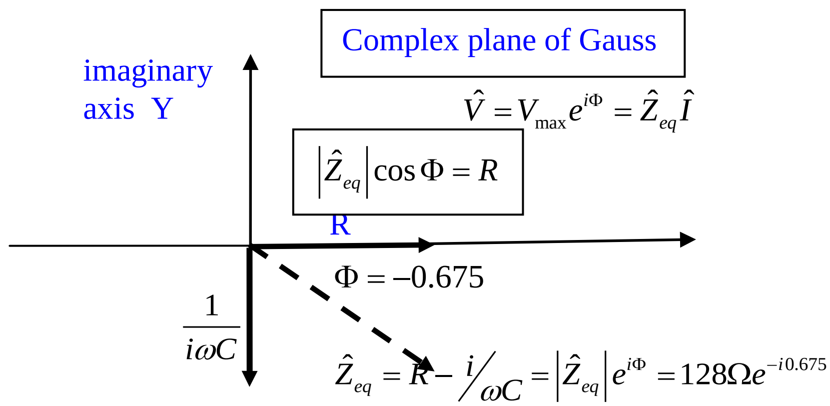
If we choose values of the preceding example with an added resistance:
$R=100\Omega;\ \omega=500/s;\ \varepsilon_{\max}=150V;\ C=25\mu F$

$$Z=\sqrt{100^2+80^2}=128\Omega;\ \tan^{-1}\frac{-80}{100}=\Phi=-0.675=-38.7°$$

$$I_{\max}=\frac{150V}{128\Omega}=1.17A$$

We get
The current is out of phase with the voltage.

(33.42) $$I(t)=1.17\sin(500t+0.675)A$$



This example illustrates the essential concepts of this whole chapter. We can also easily see how this looks in the complex plane, in terms of phasors.
All we have to find in each analysis of a circuit is the magnitude of the equivalent impedance $\left|\hat{Z}_{eq}\right|$, and its phase Φ.
Then we can calculate the current:

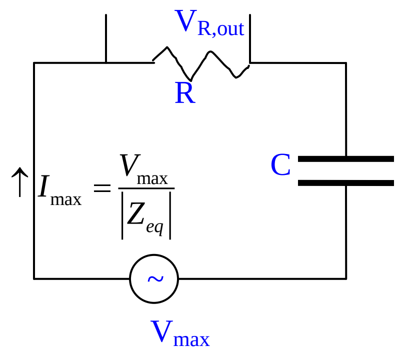
(33.43) $$\hat{I}(t)=\frac{\hat{V}(t)}{\hat{Z}_{eq}}=I_{\max}e^{i(\omega t-\Phi)}$$
$$I_{\max}=\frac{V_{\max}}{\left|\hat{Z}_{eq}\right|}$$

If the applied voltage is a sine function: $I(t)=I_{\max}\sin(\omega t-\Phi)$

If the applied voltage is a cosine function: $I(t)=I_{\max}\cos(\omega t-\Phi)$

The phase shift can be a positive or negative number.
The phase shift plays a role in the calculation of the internal power delivered (lost) to the circuit.

33.4a Taking voltages across the resistor in this RC-circuit:
If we take the voltage across the resistor in an RC-circuit, we can simply calculate that voltage by our known rules: The output voltage is equal to the resistance times the current in the circuit:



If we take the voltage across the resistor we get for the maximum voltage:

(33.44) $$\Delta\hat{V}_{R,out} = R\hat{I} \Rightarrow \Delta V_{R,\max} = R\frac{\varepsilon_{\max}}{\sqrt{R^2 + 1/(\omega C)^2}} = \frac{\omega CR\varepsilon_{\max}}{\sqrt{(\omega CR)^2 + 1}}$$

For ωCR=0.1 9.95% of the voltage passes through, for ωCR=1 we see that $\Delta V_{R,\max} = \frac{\varepsilon_{\max}}{\sqrt{2}}$. For ωCR=10 $\frac{10\varepsilon_{\max}}{\sqrt{101}} = 99.5\%\varepsilon_{\max}$ For a given CR high frequencies pass through easier than low frequencies. A setup like this which allows higher frequencies to pass through a circuit easier than lower frequencies is called a **high-pass filter**.

33.5 Power loss in an ac-circuit; rms values:

As we have now introduced a resistor we can also calculate the powerloss in any ac circuit. All ac circuits that include capacitors and/or coils have the characteristic that the current is out of phase with the voltage. This phase shift modifies the power loss to resistors. Without a resistor there is no power loss.
Calculating the power loss of an ac circuit allows us also to introduce the concept of **rms current and voltage**, because we have to calculate the average power loss for a complete cycle.

33.5a Average power loss in a circuit with resistors only:

Let us first calculate the power loss for a circuit with just a resistor. Current and voltage are in phase with each other and vary as $\sin\omega t$. The **instantaneous** real power in such a circuit is

(33.45) $$\boxed{P(t) = I \cdot \Delta V = R \cdot I^2 = I_{\max} \cdot \Delta V_{\max} \cdot \sin^2 \omega t}$$

We need to find the **average power** delivered to this circuit during one or many complete cycles. For one complete period we have:

(33.46) $$T = \frac{2\pi}{\omega}$$

The average value of function over an interval of time is defined as :

(33.47) $$f_{av} = \frac{1}{T}\int_0^T f(t)dt$$

We get

(33.48) $$\frac{1}{T}\int_0^T \sin^2\omega t \cdot dt = \frac{1}{2\pi}\int_0^{2\pi} \sin^2\theta \cdot d\theta = \frac{1}{2}$$

(33.49) Math review:

$$e^{in\theta} = \cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n \text{ for n=2} \Rightarrow$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta); \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

For the average power this means that :

(33.50) $$\overline{P} = I_{\max}\cdot\Delta V_{\max}\frac{1}{2\pi}\int_0^{2\pi}\sin^2\theta\cdot d\theta = I_{\max}\cdot\Delta V_{\max}\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}(1-\cos 2\theta)\cdot d\theta =$$
$$I_{\max}\cdot\Delta V_{\max}\frac{1}{2\pi}\frac{1}{2}(2\pi - 0) = \frac{I_{\max}\cdot\Delta V_{\max}}{2}$$

(33.51) $$P_{av} = I_{\max}\Delta V_{\max}\frac{1}{2} = \frac{I_{\max}}{\sqrt{2}}\frac{\Delta V_{\max}}{\sqrt{2}} = I_{rms}\cdot\Delta V_{rms}$$

where we made use of the concept of **root-mean-square, rms**. The average value of an alternating current or voltage is obviously 0. We customarily use the rms values instead. In ac discussions and **measuring instruments** we use these rms values, which are the values averaged over one period.

When we are using 110 Volts of ac in a household, for example, we are using a rms voltage which has a maximum value of

(33.52) $$\Delta V_{\max} = \sqrt{2}\cdot\Delta V_{rms} = \sqrt{2}\cdot 110 \text{ Volts} = 1.41\cdot 110 = 155\text{Volts}$$

Similarly for currents:

(33.53) $$\Delta V_{rms} = \frac{\Delta V_{\max}}{\sqrt{2}} = 0.707\Delta V_{\max}; I_{rms} = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max}$$

We saw in the previous chapter **any** power loss in an RLC circuit always occcurs as $-RI^2$.

Why there is no average power lost in a purely capacitive

We now can easily understand why in any purely capacitive circuit the average power is always 0.

The average power involves an integral over the product between the instantaneous voltage and current in a circuit. In purely capacitive circuits current and voltage are out of phase by $\pi/2$. This means that we always deal with the product between a cosine function and sine function without any phase shift, averaged over a complete period. This integral is always 0.

(33.54) $$\frac{1}{T}\int_0^T \sin\omega t\cdot\cos\omega t\cdot dt = \frac{1}{2\pi}\int_0^{2\pi}\sin\theta\cdot\cos\theta\cdot d\theta = 0$$

If we have an ac circuit with a combination of resistors and capacitors, the current is out of phase from the voltage as we have just seen. (The same is true for ac circuits involving resistors and coils.)

33.6 Power in an AC Circuit with current and voltage out of phase:
The current in an ac circuit is out of phase with the voltage supplied.
If $V(t)=V_{\max}\sin\omega t$ then $I(t)=I_{\max}\sin(\omega t-\Phi)=\frac{V_{\max}}{|\hat{Z}_{eq}|}\sin(\omega t-\Phi)$

The average power becomes:

(33.55) $$P_{av}=\frac{1}{T}\int_0^T\Delta V(t)I(t)dt=\frac{1}{T}\Delta V_{\max}I_{\max}\int_0^T\sin\omega t\cdot\sin(\omega t+\Phi)dt$$

We expand

(33.56) $$\sin(\omega t+\Phi)=\sin\omega t\cos\Phi-\sin\Phi\cos\omega t$$

(33.57) $$P_{av}=\frac{1}{T}\Delta V_{\max}I_{\max}\int_0^T(\sin\omega t\cos\Phi-\sin\Phi\cos\omega t)\sin\omega t dt=$$
$$\frac{1}{T}\Delta V_{\max}I_{\max}\int_0^T(\sin\omega t\sin\omega t\cos\Phi-\sin\Phi\cos\omega t\sin\omega t)dt=\frac{1}{T}\frac{\Delta V_{\max}I_{\max}}{2}\cos\Phi$$

Note that it does not make any difference whether we use a positive or negative phase shift . The integral over the mixed trig terms is 0.

Thus the average power delivered to the ac circuit is equal to:

(33.58) $$\bar{P}\equiv P_{av}=\Delta V_{\max}I_{\max}\frac{1}{2}\cos\Phi=\Delta V_{rms}I_{rms}\cos\Phi=RI_{rms}^2=\frac{|\hat{Z}_{eq}|I_{\max}^2}{2}\cos\Phi$$

$\cos\Phi$ is called the power factor; $|\hat{Z}_{eq}|\cos\Phi=R$

Now let us proceed to find the complex impedance $\hat{Z}_L$ for a coil.

33.7 Inductors in an AC circuit:
If we look at the relationship between the applied ac voltage $\hat{\varepsilon}(t)$ and the current $\hat{I}(t)$ inside of a circuit consisting only of an inductor we see that the **voltage across the coil** is proportional to the negative derivative of the current. We recall that the voltage across a coil is given by the induced emf:

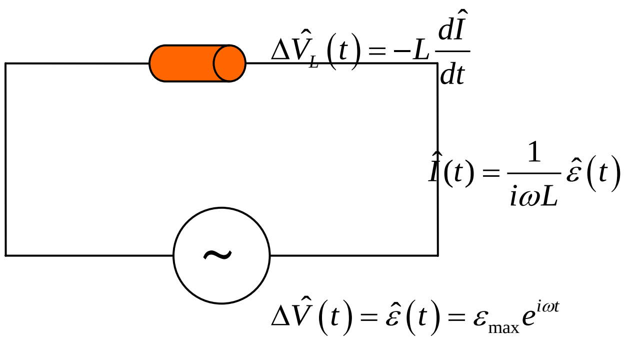
(33.59) $$\Delta\hat{V}_L(t)=-L\frac{d\hat{I}}{dt}$$

For the whole loop we use Kirchhoff's law:

(33.60) $$\underbrace{\hat{V}(t)}_{\varepsilon_{\max}e^{i\omega t}}+\Delta\hat{V}_L(t)=0\Rightarrow\hat{\varepsilon}(t)-L\frac{d\hat{I}}{dt}=0$$

Thus, the relationship between the applied ac voltage and the current in the inductive circuit can be obtained through integration of (33.60)

(33.61)
$$\boxed{\begin{aligned} &d\hat{I}(t)=\frac{\hat{\varepsilon}(t)}{L}dt=\frac{\varepsilon_{\max}}{L}e^{i\omega t}dt\Rightarrow \\ &\hat{I}(t)=\frac{\varepsilon_{\max}}{i\omega L}e^{i\omega t}=\frac{1}{i\omega L}\hat{\varepsilon}(t)\end{aligned}}$$



Multiplying equation (33.61) by the impedance $i\omega L$ we get:

(33.62)
$$\boxed{\hat{\varepsilon}(t)=\hat{V}(t)=i\omega L\hat{I}(t)}$$

Memorize: the complex inductive impedance $\hat{Z}_L$ is:

(33.63)
$$\boxed{\hat{Z}_L=i\omega L}$$

We write the complex impedance in terms of a magnitude and an exponential :

(33.64)
$$\hat{Z}_L=i\omega L=e^{i\frac{\pi}{2}}\omega L$$

(33.65)
$$\boxed{\Delta\hat{V}=\hat{Z}_L\hat{I}(t)=i\omega L\cdot\hat{I}(t)=e^{i\frac{\pi}{2}}\omega LI_{\max}e^{i\omega t}=\omega LI_{\max}e^{i(\omega t+\frac{\pi}{2})}}$$

We see that the voltage leads the current. As the outside voltage is given, we express the current in terms of the voltage:

(33.66)
$$\hat{I}(t)=\frac{\hat{\varepsilon}(t)}{\hat{Z}_L}=\frac{\hat{\varepsilon}(t)}{i\omega L}=\frac{\varepsilon_{\max}}{\omega L}e^{i(\omega t-\frac{\pi}{2})}$$

The current lags behind the applied emf. Let us study now an LR circuit.

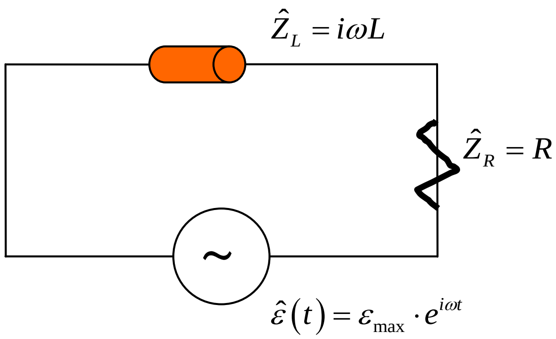
33.8 Resistor and inductor in series: LR-circuit:

For example, let us say we have a resistor (R=30Ω) **and** a coil (L=0.900H) in series; ΔV_{max}=10.0V, ω=100/s. We apply Kirchhoff's loop rule

(33.67)
$$\begin{aligned} &\hat{\varepsilon}(t)+\Delta\hat{V}_L+\Delta\hat{V}_R=0\Rightarrow\hat{\varepsilon}(t)=i\omega L\hat{I}(t)+R\hat{I}(t)=0 \\ &\hat{\varepsilon}(t)=\underbrace{(i\omega L+R)}_{\hat{Z}_{eq}}\hat{I}(t)\end{aligned}$$

The equivalent impedance is the factor in front of the complex current:

(33.68) $$\boxed{\hat{Z}_{eq} = R + i\omega L}$$



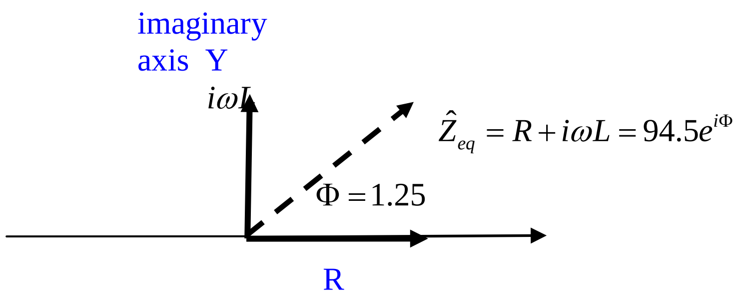
we write our equivalent impedance $\hat{Z}_{eq}$ in exponential form :

(33.69) $$\hat{Z}_{eq} = R + i\omega L = \underbrace{\sqrt{R^2 + (\omega L)^2}}_{|z|=\text{magnitude of z}} e^{i\Phi} = \text{with} \left| \begin{array}{l} \tan\Phi = \dfrac{\omega L}{R} \\ \underbrace{\Phi = 1.25 = 71.2°}_{\text{phase between V and I}} \end{array} \right.$$

(33.70) $$I_{\max} = \frac{V_{\max}}{Z_{eq}} = \frac{V_{\max}}{\sqrt{R^2 + (\omega L)^2}} = \frac{10V}{94.5\Omega} = 0.106A; I_{rms} = 0.707\,\mathrm{I}_{\max} = 74.8mA$$

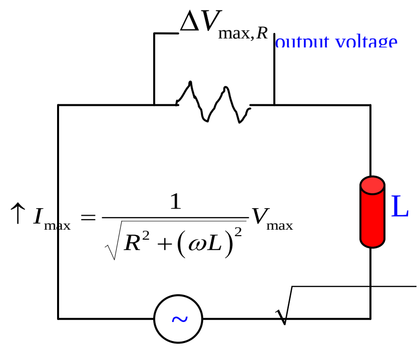
The current lags behind the voltage by the phase of 1.25rad or 71.2°.
Here are the details :(33.71)

$$\boxed{\hat{I}(t) = \frac{1}{|\hat{Z}_{eq}|} e^{-i\Phi} \hat{\varepsilon}(t) = \frac{\varepsilon_{\max}}{|\hat{Z}_{eq}|} e^{i(\omega t - \Phi)} \rightarrow I(t) = \underbrace{\frac{\Delta V_{\max}}{\sqrt{R^2 + (\omega L)^2}}}_{I_{\max}} \sin(\omega t - \Phi); \tan\Phi = \frac{\omega L}{R}}$$



33.8a Taking voltages across an element in an ac-circuit; Low pass filter:

Let us next find the maximum voltage across the resistor in this RL-circuit :



$$\Delta V_{\max,R} = RI_{\max} = \frac{R\varepsilon_{\max}}{\sqrt{R^2+\omega^2 L^2}} = \frac{\varepsilon_{\max}}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}}$$

$\varepsilon_{\max}$ input voltage

We can see that the value of the inductive impedance regulates the amount of voltage across the resistor in such a circuit. This shows how we can filter the voltage.
If ωL=0.1R 99.5% of the driving voltage passes through. If ωL=R $\Delta V_{\max,R} = 0.707\Delta V_{\max} = \Delta V_{rms}$
If ωL=10R, only 9.95% of the original voltage passes through.
This is why such a circuit which lets through more current at lower frequencies is called a **low-pass filter.**

33.9 Summary Rules:

So, to sum up: when we analyse a circuit and want to find the currents in the various branches, we apply Kirchhoff's rules and use the complex impedances for the voltage drops. We always get a relationship like:

(33.72) $$\boxed{\hat{V} = \hat{Z}_{eq}\hat{I}}$$

Combine any **impedances in series** to the **equivalent impedance** $\hat{z}_{eq}$ by using the **sum of the impedances**, $\hat{Z}_{eq} = \sum_k \hat{Z}_k$; $\hat{Z}_k$ are the complex impedances of resistors, capacitors, or coils.

Combine **impedances in parallel** according to :

(33.73) $$\frac{1}{\hat{z}_{eq}} = \sum \frac{1}{z_k} \Rightarrow \text{for two impedances: } \hat{Z}_p = \frac{1}{\frac{1}{\hat{Z}_1}+\frac{1}{\hat{Z}_2}} = \frac{\hat{Z}_1\hat{Z}_2}{\hat{Z}_1+\hat{Z}_2}$$

(33.74)

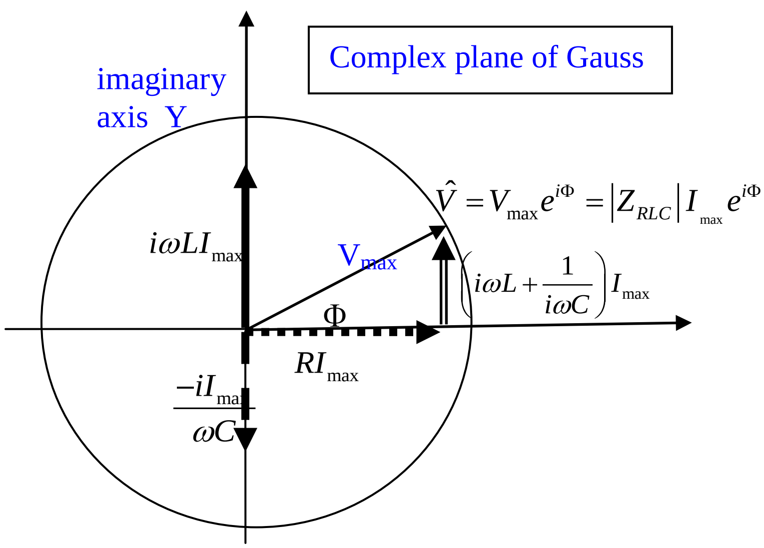
Impedance of a simple resistor $Z_R = R$
Impedance of an inductor : $\hat{Z}_L = i\omega L$
Impedance of a capacitor: $\hat{Z}_C = \frac{1}{i\omega C}$

(33.75)
$$\hat{\varepsilon}(t) = \hat{V} = \hat{Z}_{eq}\hat{I};\ \hat{I} = \frac{\hat{\varepsilon}(t)}{\hat{Z}_{equ}} = \frac{\varepsilon_{max}}{\left|\hat{Z}_{equ}\right|}e^{i(\omega t-\Phi)} = I_{max}e^{i(\omega t-\Phi)};\Phi = \tan^{-1}\left(\frac{Y}{X}\right)$$
$$\hat{Z}_{equ} = (X+iY);\left|\hat{Z}_{equ}\right| = \sqrt{X^2+Y^2}$$

This is essentially all you need to know. √Now we are ready to combine any number of passive elements in ac circuits and calculate the currents using Kirchhoffs rules and the results of the complex impedances just obtained :

33.10 RLC in series in an ac driven circuit :
This method is really a simplified and practical way of solving a differential equation of a damped oscillation driven by an outside voltage $V_{max}\sin\omega t$ .Applying Kirchhoff's rules to such a RLC circuit we get:

(33.76)
$$\hat{\varepsilon}(t) = \Delta V_R + \Delta\hat{V}_L + \Delta\hat{V}_C = R\hat{I} + i\omega L\hat{I} + \frac{1}{i\omega C}\hat{I} = \left(R + i\omega L - \frac{i}{\omega C}\right)\hat{I}$$
$$\hat{\varepsilon}(t) = R + i\left(\omega L - \frac{1}{\omega C}\right)\hat{I}(t)$$



From equation (33.76) we see that the voltage is the sum of several complex terms on the right consisting of a number multiplied by the current. If this current is represented by the complex function $I_{max}e^{i\omega t}$ we can easily represent all terms in a Gaussian complex plane, which is the same as the so-called phasor diagrams. **If we put the current $\hat{I} = I_{max}e^{i0}$ in the horizontal direction**, by choosing t =0, we find the relationship of the remaining complex amplitudes by either using addition in the complex plane or by treating the voltages as two component vectors. Any imaginary portion corresponds to a vector in the y direction and any real portion correponds to a vector in the x-direction. We readily see that the resultant vector V_{max} corresponds to the vector sum of the three R, L, C vectors. It makes an angle Φ with the vector for the current I_{max}.
Example 1:
Assume the following values: $\omega = 300/s; L = 150mH; R = 85\Omega; C = 22.0\mu F; \varepsilon_{max} = 150V$

$$\frac{1}{\omega C}=152\Omega; \omega L=45\Omega; \omega L-\frac{1}{\omega C}=-107\Omega$$
$$\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}=137\Omega; \tan\Phi=\frac{-107}{85}; \Phi=-0.899=-52°$$
$$I_{\max}=\frac{150V}{137\Omega}=1.09A; \hat{I}(t)=1.09Ae^{i(300t+0.899)}$$

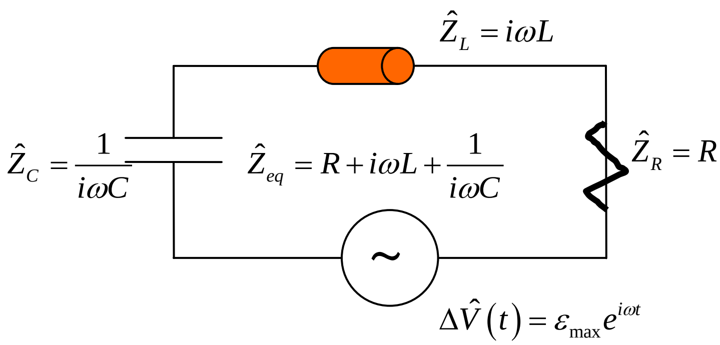
Example 2: for a AC power source of 120V, a resistor of 75 Ohms, an inductor of 25mH, a frequency of 500Hz we got: $Z_{eq}=109\Omega, \tan\Phi=\frac{\omega L}{R}; \Phi=1.047=46.3°$

The current is $\frac{\varepsilon_{\max}}{\left|\hat{Z}_{eq}\right|}=\frac{120V}{109\Omega}=1.10A$ The average power delivered to the circuit is therefore:

$$RI^2\cos\phi=75\Omega\cdot(1.10A)^2\cos 46.3°=62.8W$$

33.11 Output voltages across the resistor, capacitor, or coil:

We have a RLC circuit with a driving powersource of 150V



$$R=425\Omega; L=1.25H; C=3.50\mu F; \omega=377s^{-1}; \varepsilon_{\max}=150V$$

(33.77) $$\hat{I}(t)=\frac{\hat{V}(t)}{\underbrace{(R+i\omega L+\frac{1}{i\omega C})}_{\hat{Z}_{equ}}}=I_{\max}e^{i(\omega t-\Phi)}$$

Determine the inductance (inductive reactance), the capacitance (capacitive reactance), and the equivalent **complex impedance** of the circuit.
Inductance ωL=471Ω; capacitance=1/ωC =758 Ω

(33.78) $$\hat{z}_{eq}=R+i\omega L+\frac{1}{i\omega C}=R+i\left(\omega L-\frac{1}{\omega C}\right)=425+i(471+758)=425+i1230$$

The magnitude of the impedance is :

(33.79) $$\left|\hat{Z}_{eq}\right| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 513\Omega$$

We get the maximum current:

(33.80) $$I_{\max} = \frac{\Delta V_{\max}}{\left|\hat{Z}_{eq}\right|} = \frac{150V}{513\Omega} = 0.292A$$

The phase angle between current and voltage is given by Φ:

(33.81)
$$\Phi = \tan^{-1}\frac{Y}{X} = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) = \tan^{-1}\left(\frac{471-758}{425}\right) = \tan^{-1}(-0.6753) = -0.594 radians = -34°$$

(33.82) $$\hat{I}(t) = 0.292e^{i(\omega t + 0.594)}A$$

The **voltage across the capacitor** is obtained by multiplying the current in the circuit with the capacitive impedance $\frac{1}{i\omega C}$:

(33.83)
$$\Delta\hat{V}_C = \hat{Z}_C\hat{I} = \frac{1}{i\omega C}\hat{I} = \frac{1}{\omega C}\hat{I}e^{-i\frac{\pi}{2}} = \frac{I_{\max}}{\omega C}e^{i\left(\omega t + 0.594 - \frac{\pi}{2}\right)}$$
$$\Delta V_{C.\max} = \frac{1}{\omega C}I_{\max} = 758\Omega \cdot 0.292A = 221V$$

The voltage across the coil is

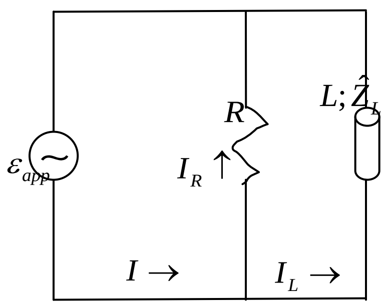
(33.84)
$$\Delta\hat{V}_L = \hat{Z}_L\hat{I} = i\omega L\hat{I} = \omega L\hat{I}e^{i\frac{\pi}{2}}$$
$$\Delta V_{L\max} = \omega L \cdot I_{\max} = 471\Omega \cdot 0.292A = 138V$$

Note that voltages and currents are out of phase with the current across the capacitor (-π/2) and across the coil (+π/2).

The average power across the capacitor or coil is $\left|\hat{Z}_{eq}\right|\Delta V_{C,rms}I_{rms}\cos\Phi$ =0

33.12 AC circuit with passive elements in parallel:

R=75.0Ω, L=25mH, V_{rms}=120V;f=500Hz=3.1416E3/s



$L; \hat{Z}_L = i\omega L$

The instantaneous voltage across R and L is the same.

$$I_R = R\Delta V_{app} = R\varepsilon_{app}$$
$$\hat{I}_L = i\omega L\hat{\varepsilon}_{app}$$

$$\hat{V}(t) = \hat{Z}_{eq}\hat{I}; \hat{Z}_{eq} = \frac{\hat{Z}_L R}{R+\hat{Z}_L} = \frac{i\omega LR}{R+i\omega L}; \hat{I} = \frac{\hat{\varepsilon}}{\hat{Z}_{eq}}; \frac{1}{\hat{Z}_{eq}} = \frac{R+i\omega L}{i\omega LR} = \frac{\omega L - iR}{\omega LR}$$

$$I_{\max} = \frac{\varepsilon_{\max}}{\left|\hat{Z}_{eq}\right|}; \frac{1}{\left|\hat{Z}_{eq}\right|} = \frac{\sqrt{(\omega L)^2 + R^2}}{\omega LR} = \frac{0.01844}{\Omega}; \left|\hat{Z}_{equ}\right| = 54.24\Omega; I_{rms} = \frac{120}{54.2}A = 2.21A$$

$$I_{\max} = 3.12A$$

The currents in the two branches are:

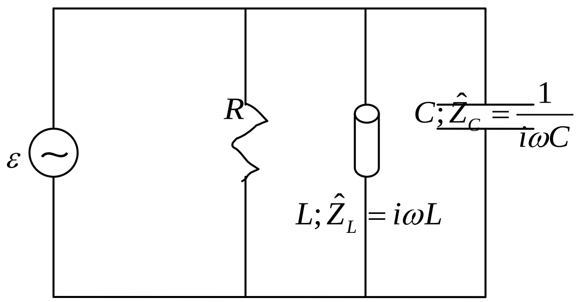
(33.85) $$\hat{I}_R = \frac{\hat{V}(t)}{R} \text{ and } \hat{I}_L = \frac{\hat{V}(t)}{i\omega L} = \frac{\varepsilon_{\max}}{\omega L}e^{i(\omega t - \frac{\pi}{2})}$$

Their maximum values are $I_{\max,R} = 2.26A$; $I_{\max,L} = 2.16A$ However, the current in the inductor branch is out of phase by π/2 with the current in the resistor branch. One is a cosine function, the other is a sine function: $\hat{I}_L = I_{\max,L}\cos\omega t; \hat{I}_R = I_{\max,R}\sin\omega t$. That is why the sum of their currents does not add up to the current in the main branch. We have to add the currents as complex functions:

(33.86) $$\hat{I} = \hat{I}_L + \hat{I}_R = \frac{\hat{\varepsilon}(t)}{i\omega L} + \frac{\hat{\varepsilon}(t)}{R} = \hat{\varepsilon}(t)\left(\frac{1}{R} + \frac{1}{i\omega L}\right) = \varepsilon_{\max}e^{i\omega t}\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2}e^{i\Phi};$$
$$\Phi = \tan^{-1}\left(\frac{-1/\omega L}{1/R}\right) = \tan^{-1}\left(-R/\omega L\right)$$

(33.87) $$I_{\max} = \sqrt{(2.26)^2 + (2.16)^2} = 3.12A$$

Let us next look at a circuit where **all three components are in parallel**.We find the equivalent impedance by adding the components in parallel:



(33.88) $$\frac{1}{\hat{Z}_{equ}} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C = \frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right); \hat{I} = \frac{\hat{\varepsilon}}{\hat{z}_{eq}}$$

It is easier to leave the expression like this rather than solving for $\hat{z}_{eq}$

(33.89) $$\frac{1}{\hat{Z}_{equ}} = \frac{1}{R} - i\left(\frac{1}{\omega L} - \omega C\right) = \underbrace{\sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}}_{\frac{1}{|Z_{equ}|}} e^{-i\Phi}$$

$$\tan\Phi = \frac{\frac{1}{\omega L} - \omega C}{\frac{1}{R}}$$

We see that the maximum current is given by:

(33.90) $$I_{\max} = \left|\frac{1}{\hat{Z}_{equ}}\right| \varepsilon_{\max} = \underbrace{\sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}}_{\frac{1}{|Z_{equ}|}} \varepsilon_{\max}$$

The voltage differences across all three elements are the same. The currents are, respectively:

(33.91)

$$I_R = \frac{\varepsilon_{app}}{R};$$

$$\hat{I}_C = \frac{\hat{\varepsilon}_{app}}{\frac{1}{i\omega C}} = i\omega C\hat{\varepsilon}_{app}$$; current leads by $\frac{\pi}{2}$ over the applied voltage

$$\hat{I}_L = \frac{\hat{\varepsilon}_{app}}{i\omega L} = -i\frac{\hat{\varepsilon}_{app}}{\omega L}$$; current lags by $\frac{\pi}{2}$ with reference to the applied voltage

33.12 Resonance in a Series RLC Circuit.

The maximum current in a series RLC circuit is given by

(33.92) $$I_{\max} = \frac{\Delta V_{\max}}{|z_{eq}|};\ with\ |z_{eq}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

We get a maximum value for the current when the equivalent impedance is smallest, which is the case for :

(33.93) $$\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC}$$

The corresponding frequency is called the resonance frequency:

(33.94) $$\omega = \frac{1}{\sqrt{LC}}$$

At the resonance frequency we have for the current in the circuit :

(33.95) $$I_{\max} = \frac{\Delta V_{\max}}{R}$$

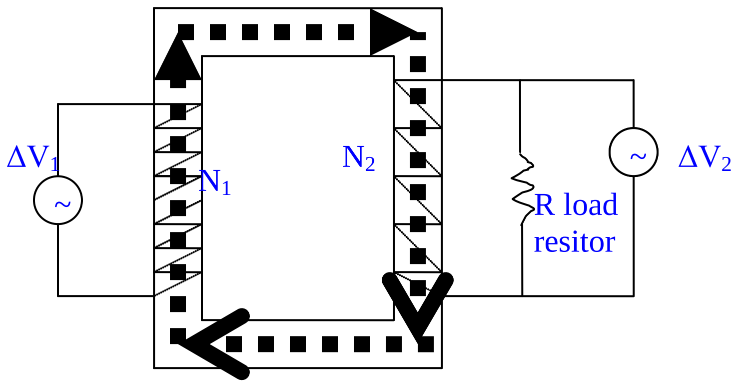
33.14 Transformers and Power Transmission.

A transformer consists essentially of a core of soft ferromagnetic iron, which increases any magnetic field inside it by a factor of a few hundred. There are two coils wound around this iron core, one, the primary coil is connected to an ac power source, which creates a huge magnetic field inside of the first set of N_1 coils. This primary circuit is essentially a **purely inductive circuit, with negligible resistance**. It induces a time varying magnetic field. The voltage between its terminals are given by Faraday's law:

(33.96) $$\Delta V_1 = -N_1 \frac{d\Phi_1}{dt}; \Phi_1 = A_1 B_1(t)$$

This same field also passes through the secondary coil with N_2 turns, creating a secondary voltage across its coil. This secondary circuit is connected to a load resistance R and a switch:

(33.97) $$\Delta V_2 = -N_2 \frac{d\Phi_1}{dt}; \Phi_1 = A_1 B_1(t)$$



If the magnetic field through each coil is exactly the same, the **flux through one coil surface** is also the same, the only difference is in the number of coils. By dividing the two equations we get:

(33.98)
$$\boxed{\frac{\Delta V_1}{\Delta V_2} = \frac{N_1}{N_2} \Rightarrow \Delta V_2 = \frac{N_2}{N_1}\Delta V_1}$$

Depending on the ratio between the number of coils we can get a smaller or a larger secondary voltage, hence the name transformer.
When the switch in the secondary circuit is closed, a current I_2 is induced in the secondary. If the load in the secondary circuit is a pure resistance, the induced current I_2 is in phase with the induced voltage V_2. The power supplied to the secondary circuit must be supplied by the ac source connected to the primary circuit. In an ideal transformer where there are no losses (5 to 1 % losses are typical.) the power supplied by the source is equal to the power in the secondary circuit.

(33.99)
$$I_1\Delta V_1 = I_2\Delta V_2$$

The value of the load resistance R_L determines the secondary current:

(33.100)
$$I_2 = \frac{\Delta V_2}{R_L}$$

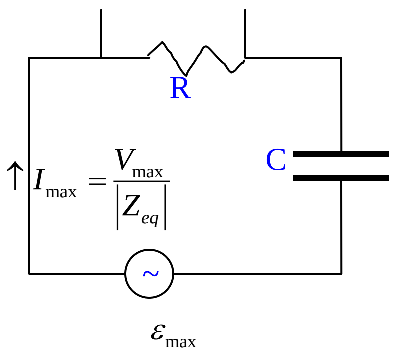
Example:A transmission line has a resistance per unit length of 4.50E-4Ohms/m. It is to be used to transport 5.00MW over 500km. The output voltage of the generator is 4.50kV. What is the line loss if the voltage is stepped up to 500kV? What is the cost due to heat during one day if 1kWhr costs 10cents?

The current in the wire is 10A=Power/voltage. The resistance in the wire is R=225Ω, therefore the powerloss is RI^2=22.5kW. During 24 hours the wire uses 22.5*24 kWhrs in energy, which costs $54.

If one would try to transmit the power at the voltage of the generator the current in the wire would be:1.11kA, which would generate RI^2 in heat. Per meter this would be 556Watts. This would melt the wire in short order. If the wire has a mass of copper of 1g/m, how long would it take to get it to melt? c for copper is 0.0924 cal/gC°. Melting point is 1083°C. I takes 100cal or 418 Joules to get to the melting point of copper for one gram, providing that the resistance of copper does not increase, which of course it does. This means it does not take even 1 second to get to th melting point, 0.75s. (The latent heat of fusion for copper is 134 J/g.) The resistivity of copper is 1.7E-8 Ohms/m. The temperature coefficient is 3.9E-3/C°
Remember that the resistance increases with temperature as $R = R_0 e^{\alpha(T-T_0)}$

<u>33.15 Rectifiers and Filters; refer to 33.7a and 33.4a</u> :
Many household devices require low voltage dc currents. They come with transformers, which, plugged into a 120 Volt ac outlet, transform the current down to typically 12 volt ac. In a second step the ac voltage gets changed into dc voltage by means of diodes. These diodes allow current to pass through only in one direction, thus cutting off the negative portion of a sine or cosine

wave. The resulting half wave sine curve can be smoothed out by adding a capacitor to the circuit. Remember that a discharging LRC circuit corresponds to a damped oscillation. Thus after the capacitor has been charged and it discharges through the resistor, the downward sine curve is overpowered by the exponential decrease of the charge according to $e^{-\frac{t}{RC}}$. If we choose the RC value in such a way that it is equal to approximately T, the downward sine curve will just be an exponentially decreasing curve until the next upward (charging) branch of the sine curve is reached. In this way the sine curve is smoothed out to an almost constant current. At the next step certain frequencies need to be filtered out, which means that the amplitude of the voltages corresponding to the unwanted frequencies should be made small. Just like the LR circuit discussed earlier, a CR circuit can be designed to filter out certain frequencies.



For the current in the circuit we get:

(33.101)
$$\Delta\hat{V} = \hat{Z}_{eq}\hat{I} = \left(R + \frac{1}{i\omega C}\right) = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}\,\hat{I}e^{i\Phi};$$
$$\hat{I} = \frac{\varepsilon_{max} e^{i(\omega t - \Phi)}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

If we take the voltage across the resistor we get:

(33.102)
$$\Delta\hat{V}_R = R\hat{I} \Rightarrow \Delta V_{R,max} = \frac{R\varepsilon_{max}}{\sqrt{R^2 + 1/(\omega C)^2}} = \frac{\omega CR\varepsilon_{max}}{\sqrt{(\omega CR)^2 + 1}}$$

For ωCR=0.1 9.95% of the voltage passes through, for ωCR=1 we see that $\Delta V_{R,max} = \Delta V_{rms} = 0.707\Delta V_{max}$. For ωCR=10 $\frac{10\Delta V_{max}}{\sqrt{101}} = 99.5\%\Delta V_{max}$ For a given CR high frequencies pass through easier than low frequencies. We can therefore build high pass filters with CR circuits.