Chapter 31. Faraday's law

(31.1)
$$curl\vec{E} \equiv \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Leftrightarrow \oint_{\partial A} \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t} \iint_A \vec{B} \bullet d\vec{A} = -\frac{d\Phi_B}{dt} \Rightarrow$$
$$\varepsilon = -\frac{d\Phi_B}{dt}$$

1. A 50 turn rectangular coil of dimension 5x10cm is allowed to fall from a position where B=0 to a new position where B=0.5T and the magnetic field is directed perpendicular to the plane of the coil. Calculate the magnitude of the average emf induced in the coil if the displacement occurs in 0.25 s. (0.5V)
2. Explain with a formal mathematical argument why $\vec{E} = -\overrightarrow{grad} \cdot V$ and $curl\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ are incompatible, and therefore cannot refer to the same kind of electric field.

3. (3) A 25 turn circular coil of wire has a diameter of 1.00m. It is placed with its axis along the direction of the Earth's magnetic field 50E-6 T, and then, in 0.200s it is flipped 180°. An average emf of what magnitude is generated in the coil? (9.81mV)

4. (7) A strong electromagnet produces a uniform magnetic field of 1.60T over a cross-sectional area of $0.200m^2$. We place a coil of 200 turns and a total resistance of 20.0 Ohms around the electromagnet. We then smoothly reduce the current in the electromagnet until it reaches 0 in 20.0 ms. What is the current induced in the coil? (160A)

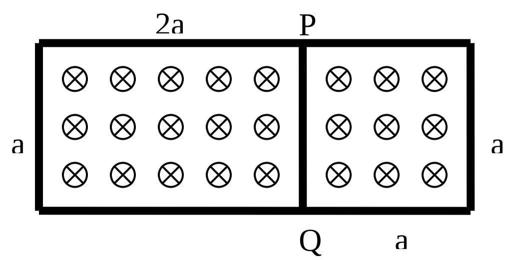
5. (8) a) A loop of wire in the shape of a rectangle of width w and length L and a long, straight wire carrying a current I lie on a table top as shown below.

Determine the magnetic flux through the loop due to the current I.
b) Suppose the current is changing with time according to I=a+bt where a and b are constants. Determine the emf that is induced in the circuit if b=10A/s, h=1.00cm, w=10.0cm, and L=100cm. What is the direction of the current in the rectangle?

$$\Phi = \frac{\mu_0}{2\pi} IL \ln\frac{(h+w)}{h}$$

b)4.80E-6V

6. (13) In the figure below find the current in the branch PQ of the circuit.

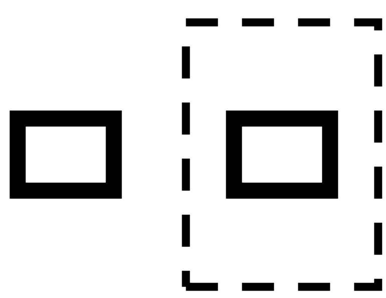


The circuit is a rectangle of sides 3a and a as indicated, a=65.0cm. It is located in a magnetic field $B(t) = (1.00E-3T/s)\cdot t$ The resistance per length of wire is 0.100 Ohms/m.
Hint, insert resistors into the branches, and emf's, then apply Kirchhoff's rules.
I_{PQ}=-0.283mA.

7. (14) A long solenoid has n=400turns/m and carries a current given by: $I(t) = 30.0A\left(1-e^{-1.60t}\right)$. Inside the solenoid and coaxial with it is a coil that has a radius of 6.00cm and consists of a total of N=250 turns of fine wire. Calculate the emf induced in the interior coil by the changing current?

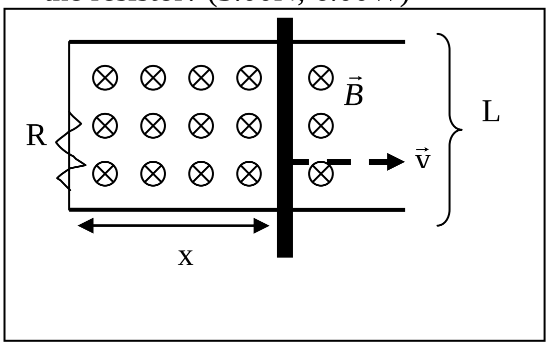
($-68.2mV \cdot e^{-1.6t}$)

8. (18) A toroid having a rectangular cross section (a=2.00cm high, b=3.00cm wide) and inner radius R=4.00cm (i.e. outer radius =4+3=7cm) consists of 500 turns of wire that carries a sinusoidal current $I = I_0 \sin\omega t$ with $I_0 = 50.0A$ and a frequency f=60.0Hz. A coil that consists of 20 turns of wire links with the toroid, see picture. Determine the emf as a function of time.



$\varepsilon = -0.422V \cdot \cos\omega t$

9. (23) The picture below shows a bar that can slide on two frictionless rails. The resistor is 6.00Ohms, and a 2.50T magnetic field is directed perpendicularly into the plane. Let L=1.20 m. Calculate the applied force necessary to move the bar to the right with a constant velocity of 2.00m/s. At what rate is energy delivered to the resistor? (3.00N; 6.00W)



10. A flexible metallic wire with linear density of 3.00g/m is stretched betweent two clamps 64.0 cm apart and held under tension 267N. A U magnet is placed around the wire such that the magnetic field of the magnet is perpendicular to the wire and produces a uniform field of strength 4.50mT over a 2.50cm length at the center of the wire. It is negligible elsewhere. The wire is set vibrating at its fundamental frequency (lowest).The section of the wire in the magnetic field moves with a uniform amplitude of 1.50cm. Find the frequency and the amplitude of the emf induced between the ends of the wire. (233Hz,1.98mV).
11. (33) A magnetic field directed into the page changes with time according to $B = (0.0300t^2 + 1.40)T$ where t is in seconds. The field has a circular cross section of radius R=2.50cm. What are the magnitude and direction of the electric field at a point r=0.0200m at t =3.00s. (E=-1.80mV/m, ccw)
12. (35) A coil of area 0.100m^2 is rotating at 60.0rps with the axis of rotation perpendicular to a 0.200T magnetic field.

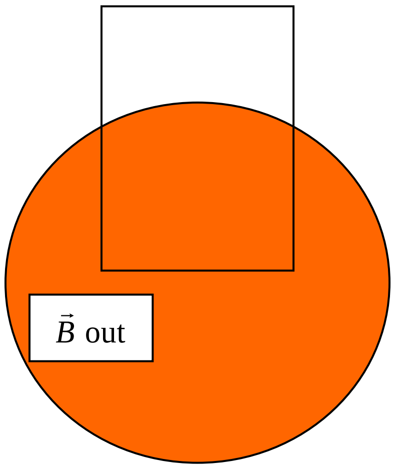
a) If the coil has 1000 turns, what is the maximum emf generated in it? b) What is the orientation of the coil with respect to the magnetic field when the maximum induced voltage occurs? (7.54kV; plane of coil is parallel to B.)

13. (39) An electro motor in normal operation carries a direct current of **0.850A** when connected to a **120 V** power supply. The resistance of the motor windings is **11.8 Ohms**. a)While in normal operation, what is the **back emf** generated in the motor?

b) At what rate is internal energy produced in the windings?
Internal energy means heat energy, which is produced in the resistor.
$Power = RI^2 = 8.53W$
c)Suppose that a malfunction stops the motor shaft (in the coil) from rotating. At what rate will internal energy be produced in that case?
The emf being 0, the current will be $I = 120V/11.8\Omega = 10.2A$
$Power = RI^2 = 1.22kW = I\Delta V$

14. (43)A conducting rectangular loop of mass M, resistance R and dimensions w by l falls from rest into a magnetic field B as shown below. During the time interval before the top edge of the loop reaches the field , the loop approaches a terminal speed v_T. Show that $v_T = \frac{MgR}{B^2 w^2}$

Why is the terminal velocity proportional to R, and inversely proportional to B^2?



15. (45 new) A proton moves through a uniform electric field given by $\vec{E} = 50.0 \frac{V}{m} \vec{j}$ and a uniform magnetic field of $\vec{B} = (0.200\vec{i} + 0.300\vec{j} + 0.400\vec{k})T$

Determine the acceleration of the proton when it has a velocity of $\vec{v} = 200\vec{i} \frac{m}{s}$

$a_y = -30 \frac{q}{m}; a_z = 60 \frac{q}{m}$