Homework: See website.

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31.1 Faraday's Law of Induction.
We saw in the last chapter that a changing electric field (or its flux) creates a changing magnetic field which is responsible for the displacement current density (or displacement current) in the space between the plates of a capacitor.

(31.1) $$curl\vec{B} = \mu_0 \left(\vec{j} + \underbrace{\varepsilon_0 \frac{\partial \vec{E}}{\partial t}}_{\vec{j}_d} \right)$$

(31.2) $$\oint_{\partial A_1} \vec{B} d\vec{s} = \mu_0 \iint_{A_1} \left(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \bullet d\vec{A} = \mu_0 I + \varepsilon_0 \mu_0 I_d$$

It is quite natural (symmetry!) to ask whether it would also be possible that changing magnetic fields would create a changing electric field and all the associated quantities like current, voltage, etc. Faraday discovered in an ingenious set-up of experiments that this is indeed the case.

He found that by simply taking a magnet and moving it around in front of an electric wire, a current was induced in that wire, without the presence of any battery.

The localized law which quantitatively describes this relationship is known as **Faraday's law**: A time-changing electric field curls around any time changing magnetic field:

(31.3) $$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Applying Stokes' theorem to this gives us **Faraday's law in integral form**:

(31.4) $$\boxed{\iint_A \vec{\nabla} \times \vec{E} \bullet d\vec{A} = \underbrace{-\iint_A \frac{\partial \vec{B}}{\partial t} \bullet d\vec{A}}_{-\frac{d\Phi_B}{dt}} = \underbrace{-\frac{d}{dt}\left(\iint_A \vec{B} \bullet d\vec{A} \right)}_{-\frac{d\Phi_B}{dt}} = \underbrace{-\frac{d}{dt}\left(\iint_A B \bullet dA \cos\theta \right)}_{-\frac{d\Phi_B}{dt}}}$$

A time-changing magnetic flux creates an emf in an adjacent conducting wire.

(31.5) $$\boxed{\iint_A \vec{\nabla} \times \vec{E} \bullet d\vec{A} = \oint_{\partial A} \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt} = emf = \varepsilon}$$

Note that the line integral over the electric field extends over a whole loop. If the electric field would derive from a potential field V, implying a conservative electric field, this integral would be 0. Therefore we conclude that the **electric field in Faraday's law is a non-conservative field**. Furthermore, note that the surface and boundary of that surface, which we use in the application of Faraday's law are not the same as we used in the application of Ampere's law. Indeed, if a time changing current creates a magnetic field according to Ampere's law, the magnetic field creates an electric field around it. The surfaces chosen are perpendicular to each other, so that we may choose to talk about an Ampereian surface and a Faraday surface.

Faraday's law of induction:

(31.6) $$\varepsilon = -\frac{d\Phi_B}{dt}$$

The flux on the right hand side can change in three different ways:

- **a changing magnetic field**, $\varepsilon = -A\frac{dB}{dt}$
- **a changing surface**, $\varepsilon = -B\frac{dA}{dt}$ Motional emf
- **a changing angle between surface and magnetic field**, $\varepsilon = -BA\frac{d\cos\omega t}{dt}$
- and any combination of these.

A changing magnetic flux will induce an emf in any conducting material, which will induce a current in any loop of wire properly positioned.

$$\mathrm{I}_{\mathrm{ind}} = \frac{\varepsilon}{R}$$

We can immediately see that we need a minus sign in Faraday's law $curl\vec{E} = -\frac{\partial \vec{B}}{\partial t}$. A time changing magnetic field creates loops of electric fields around it. If there are charges that are free to move they will feel the electric force $\vec{F} = q\vec{E}$. In a conductor this will lead to a current and consequently to an emf which is the circulation of this non-conservative electric field. In this conductor with a resistance R we have a current:

(31.7) $$I_{ind} = \frac{\oint_{loop} curl\vec{E}\cdot d\vec{s}}{R} = \frac{\varepsilon}{R}$$

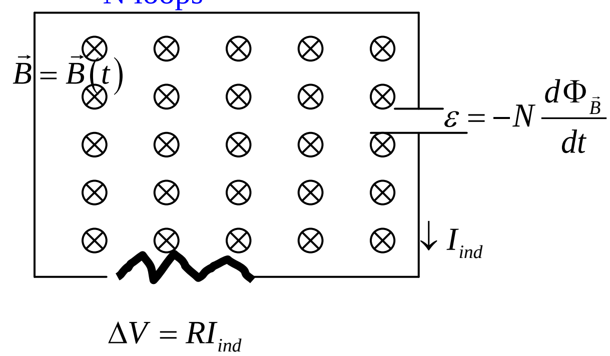
A time-changing current in a conducting loop in turn creates a magnetic field according to,

(31.8) $$curl\vec{B} = \cancel{\mu_0\vec{j}} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t} \Rightarrow \oint_{\partial A}\vec{B}\bullet d\vec{s} = \mu_0 I_d$$

This would lead to an infinite built-up of electromagnetic energy, violating energy conservation. Therefore, the induced emf (current) will be such that it must oppose the action which originally created the emf. This negative sign is explicitly stated in what is called **Lenz's law**. (See later.) **If the wire surrounding the surface consists of N loops we have to use the circulation through all of these wires, therefore producing N times the emf of one single loop**.

(31.9) $$\varepsilon = -N\frac{d\Phi_B}{dt}$$

It is often helpful to **symbolically** introduce an emf powersource in a circuit with an induced current, even if there is no physical power source in the circuit. For example, if we have a coil of N loops, surrounding a **time changing B-field**, and the coil has a resistance of R we could draw a sketch as shown below:



Let us study a few cases in which an emf is created acording to Faraday's law. **A time varying magnetic flux creates an emf**. We look at situations in which the surface through which we have a constant magnetic field varies in time.

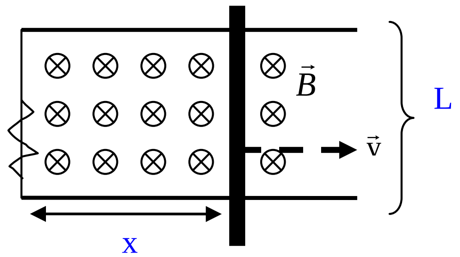
31.2 Motional emf and Power $\varepsilon = -B\frac{dA}{dt}$.

We consider a simple case of a changing surface perpendicular to a constant magnetic field. This changing area can be created, for example, by a conducting bar being dragged over two conducting rails, both being perpendicular to a constant magnetic field. The rails are connecting by a conducting wire (left) with resistance R.The area intercepting the magnetic field is given by

$L \cdot x = A \Rightarrow \frac{dA}{dt} = L\frac{dx}{dt} = L \cdot \mathrm{v}$ So we obviously have a changing magnetic flux:

$\frac{d\Phi_B}{dt} = \frac{d}{dt}(BA) = B\frac{dA}{dt} = BL\mathrm{v}$ which will induce an emf in the circuit

(31.10) $$\boxed{\varepsilon = -BL\mathrm{v}}$$



An exterior force must drag the bar to the right, counter-acting the braking force of the induced emf, current, and force acting on the current in the magnetic field.

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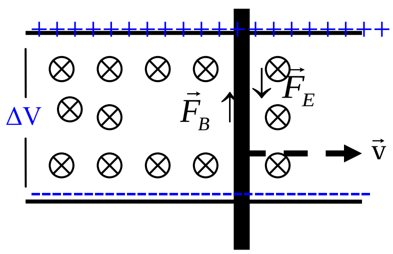
with the current (31.11) $I_{ind} = \frac{\varepsilon}{R} = -\frac{BL\text{v}}{R}$

Before we investigate that further, let us look at the same setup but understand it through the concepts of moving charges:

First, we consider the situation **without a connecting wire** between the two rails. The bar moves at constant velocity to the right on the two **frictionless** conducting rails. (We don't need an applied force as there is no work done against friction.)



While the bar moves to the right with speed v, an upward magnetic force F_B =qvB is created which pushes positive charges up, (**negative charges down**). As these charges accumulate, an electric force between the stationary surplus charges arises. This electric force F_E =qE is opposite to the magnetic force. This movement of charges continues **until an equilibrium between the upward magnetic force and the downward electrostatic force is established**.

(31.12)

$$\vec{F}_B = q\vec{\text{v}} \times \vec{B}; F_B = q\text{vB} \text{ directed from bottom to top}$$
$$\vec{F}_E = q\vec{E} \Rightarrow F_E = qE \text{ directed from top to bottom}$$
$$q\text{vB} = qE \text{ in equilibrium; E=v}B$$

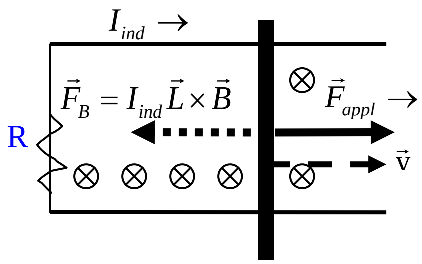
Thus, a potential difference $\Delta V = E \cdot L$ between the surplus charges on both vertical ends of the bar is established. This potential difference is maintained as long as the bar keeps moving and as long as the circuit is not completed with a connecting wire at another location.

(31.13) $$\Delta V = EL = \text{v}BL$$

Note that in the analysis of this situation we did not use Faraday's law to explain the phenomenon. However, the value of this potential difference ΔV=BLv is the same as the emf induced in a closed loop. In the moment the top and bottom rail are connected through a conducting wire with resistance R, the charges will flow around the closed loop forming a current. What will happen in that scenario follows:

Second:We connect the top with the bottom through a connecting wire with resistance R. Now, an induced current will flow through the completed loop. The potential difference across the resistor is obviously $I_{ind}\cdot R$ We now have a complete loop, so that we can talk about a changing magnetic flux through the closed circuit. However, we should not talk about a voltage difference but about an **induced emf**, and an induced current I_{ind} because

(31.14)



$$curl\vec{E} = -\frac{\partial\vec{B}}{\partial t}\neq 0 \Rightarrow \varepsilon = -\frac{d\Phi_B}{dt}$$

Remember that a vector field cannot be conservative when its curl is different from 0, or, which is the same, when its integral around a closed loop is different from 0.

The polarity of the emf is such that it opposes the process which created it in the first place. What created it is the applied force which drags the bar across the rails from left to right. If we do not have an applied force, the movement of the bar will decrease exponentially. The induced current $I_{ind}=\frac{\varepsilon}{R}=\frac{-BvL}{R}$, being situated in the constant magnetic field, produces an opposing force in the bar according to

(31.15) $$\vec{F}_B = I_{ind}\vec{L}\times\vec{B}$$

Let us apply Newton's law to this situation:

$$m\frac{dv}{dt} = I_{ind}BL \Rightarrow \frac{dv}{v}=\frac{I_{ind}L}{m}dt \Rightarrow \text{with } I_{ind} = -\frac{BLv}{R}$$

(31.16) $$\ln v = -\frac{B^2L^2}{Rm}t + c$$

$$v(t) = v_0 e^{-\frac{t}{\tau}};\ \tau = \frac{Rm}{B^2L^2}$$

Now, let us use energy considerations to derive how the velocity decreases. We have the kinetic energy of the moving bar. As it slows down, its original kinetic energy goes to 0. The change in its kinetic energy is therefore negative.

The induced current in the conducting wire decreases also and therefore the power RI^2 delivered to the resistor by the magnetic field. But, while the kinetic energy of the bar goes to 0, the energy delivered to the resistor increases (at a slowing rate) from 0 to its final value. Therefore,

(31.17) $$P=\frac{dE}{dt}=I^2R=-\frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

The induced current in the bar is the result of the induced emf (compare (31.11))

(31.18) $$|I| = \frac{BLv}{R}$$

Therefore:

(31.19) $$\frac{B^2L^2v^2R}{R^2} = -mv\frac{dv}{dt}\Rightarrow\frac{dv}{v}=-\frac{B^2L^2}{Rm}dt$$

which is the same as in (31.16).

To get an opposing magnetic force **the induced current of negative electrons in the bar must be directed downward**.

31.2a Transformation of mechanical power into electric power:

We study the energy of the system with a constant exterior force F_{app} applied:

For the bar to keep moving at constant velocity, this applied force is equal and opposite to the magnetic force on the bar $F_B = I_{ind} LB = \frac{\varepsilon}{R} LB = \frac{\mathrm{v}LB}{R} LB = \frac{\mathrm{v}L^2B^2}{R}$

Let us show that the power required to move the bar to the right is equal to the electric power generated due to Faraday's law. In other words, let us see how this setup actally corresponds to a generator of electric energy dissipated in the load. The power for a constant force is given by $power = \vec{F} \cdot \vec{\mathrm{v}}$.

(31.20) $P = \vec{F}_B \cdot \vec{\mathrm{v}} = \frac{\left(\mathrm{v}L^2B^2\right)}{R}\mathrm{v} = \frac{\left(\mathrm{v}LB\right)^2}{R} = \frac{\varepsilon^2}{R}$

The power $\vec{F}_B \cdot \vec{\mathrm{v}}$ applied by the exterior mechanical force is equal to the power delivered to the resistor $\frac{\varepsilon^2}{R}$ by the current in the loop, i.e. we confirm energy conservation.

31.2b Motional emf induced in a bar rotating in a constant magnetic field $\varepsilon = -B\frac{dA}{dt}$:

In order to have a closed circuit we must connect the two ends of the bar with a conducting wire.

We have seen that the emf in a **linearly** moving bar is $\varepsilon = -\frac{d\Phi_B}{dt} = -BL\mathrm{v}$ Now, in a **rotating** bar, every segment of length dr of the bar moves at a different velocity $\mathrm{v} = \omega r$, depending on the distance r to the center of rotation. We therefore calculate the infinitesimal emf for a small area bounded by the lengths $dr \text{ and } rd\theta$

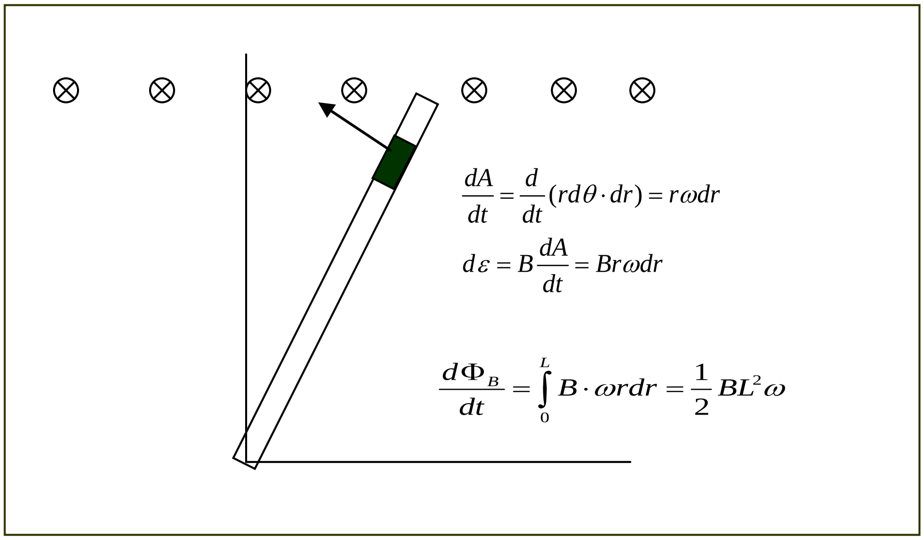
$$dA = rd\theta \cdot dr \Rightarrow$$

(31.21) $$\frac{dA}{dt} = r\frac{d\theta}{dt}dr = r\omega dr$$

This time-changing infinitesimal area times the magnetic field gives us the infinitesimal induced emf for the small segment of the bar, with length dr $d\varepsilon = B \cdot r\omega dr = Br\omega dr$

To get the total emf induced in the rotating bar of length L we must integrate over the infinitesimal portions of the emf from r=0 to r=L.

(31.22) $$\varepsilon = \int_0^L d\varepsilon = \int_0^L B \cdot \omega r \cdot dr = \frac{BL^2\omega}{2}$$



31.3 Lenz's Law.

Faraday' law includes Lenz's law as the minus sign in

(31.23) $$\varepsilon = -\frac{d\Phi_B}{dt}$$

which is the result of taking the surface integral over the differential equation (differential form of Faraday's law):

(31.24) $$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

and using Stokes theorem which yields the integral form of Faraday's law.

(31.25) $$\boxed{\iint_A \vec{\nabla} \times \vec{E} \cdot d\vec{A} = -\iint_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} \Rightarrow \oint_{\partial A} \vec{E} \cdot d\vec{s} = \varepsilon = -\frac{d\Phi_B}{dt}}$$

A changing magnetic flux induces an emf in a circuit. The polarity of the emf is such that it opposes whatever causes the original changing flux.Lenz's law explicitly states the existence of the negative sign in Faraday's law:

(31.26)

The induced current (created by a primary magnetic field) in a loop is in the direction that creates a (secondary) magnetic field which opposes the change in magnetic flux (of the primary magnetic field) through the area enclosed by the loop.

Let us just review the principles at work here:

a) a changing magnetic flux of field B_1 creates an electric emf and current in a conducting loop.

(31.27) $$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

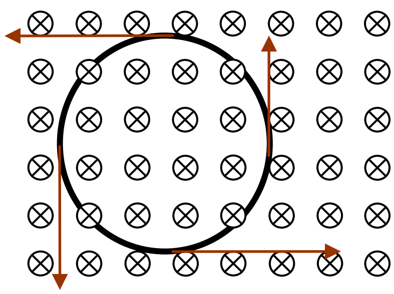
b) any current creates a magnetic field around it:

(31.28) $$\vec{\nabla}\times\vec{B}=\mu_0\vec{j}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

Without the minus sign we would have an ever increasing magnetic and electric field, which would be an ever increasing energy, which would violate the conservation of energy law.

31.4 Induced emf and Electric Fields.

Another point which must be emphasized here is that the induced emf should not be written as a voltage difference because **voltage difference implies the existence of a conservative electric field**. The electric field in (31.27) is explicitly **not conservative because** $curl\vec{E}\neq 0$ **the curl of E is not 0**. The path integral of E around a closed loop is not 0 but is equal to the emf. Remember that the voltage difference was defined as:



(31.29) $\Delta V=-\int_a^b\vec{E}\cdot d\vec{s}$

which holds only for a conservative electric field.

(31.30) $div\vec{E}=\frac{\rho}{\varepsilon_0};\vec{E}=-gradV;\vec{\nabla}\times\vec{E}=\vec{0}$

This is best illustrated by putting a circular conductor into a time-changing magnetic field. (example in 31.4)

According to $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$ we get an electric field

(31.31) $2\pi rE=-\dot{B}\pi r^2$ or

(31.32) $E(t)=-\frac{r}{2}\frac{dB}{dt}$

This electric field , created by the primary magnetic field $B_1(t)$ creates a secondary magnetic field around itself according to Maxwell's law:

(31.33) $$curl\vec{B}_2(t)=\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

We can see that the secondary magnetic field cannot be such that it would strengthen the original primary magnetic field. Lenz's law takes care of that. This is of course the effect of the minus sign in Faraday's law: $\varepsilon=-\frac{d\Phi_B}{dt}\Leftrightarrow\vec{\nabla}\times\vec{E}(t)=-\frac{d\vec{B}}{dt}$

Example for an emf created by a changing magnetic field:

Example 31.7

Calculate the induced electric field outside of a solenoid at a distance r>R. The long solenoid has a radius R and n wires per unit length. It carries an alternating current

(31.34) $$I=I_0\cos\omega t$$

Obviously, the solenoid creates a magnetic field inside $B=\mu_0 nI_0\cos\omega t$. As this magnetic field varies in time it induces an electric field everywhere around itself according to:

(31.35) $$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} = \mu_0 n I_0 \omega \sin\omega t$$

$$\oint_{\partial A} \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\iint_A \vec{B}\cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

The circulation of E around an arbitrary loop (surrounding the magnetic field) with radius r is equal to the flux of the changing magnetic field:

(31.36) $$2\pi r E = \pi r^2 \mu_0 n I_0 \omega \sin\omega t$$

If we would place a conductor at this location we would obviously create an emf

(31.37) $$\varepsilon = 2\pi r E = \pi r^2 \mu_0 n I_0 \omega \sin\omega t$$

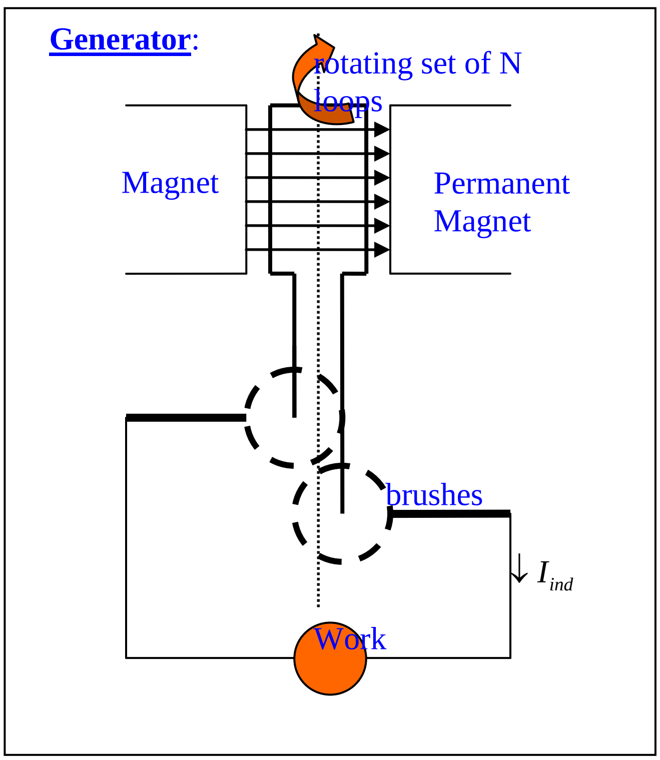
This principle is exploited in generators of electric power.

31.5 Generators and Electric Motors.

If we take a wire bent into the form of a loop and rotate it with frequency ω through a mechanical device (falling water in a dam) inside of a magnet, we obviously create an emf in that wire according to:

(31.38) $$\varepsilon = -\frac{d}{dt}\left(\vec{B}\cdot\vec{A}\right) = -\frac{d}{dt}\left(BA\cos\theta\right) = -\frac{d}{dt}\left(BA\cos\omega t\right) = BA\omega\sin\omega t$$

The emf changes signs (polarity) every 180 degrees.



Brushes take this emf and lead it into an exterior circuit, where we have the alternating current:

(31.39) $$I_{ind} = \frac{\varepsilon_{ind}}{R}$$

Here we have the source of the notion **electromotive force emf.** It is originally the ac voltage of an electric power generator delivered to a circuit. By using a commutator (a split ring conductor) we avoid reversing the polarity and create a time vaying dc-current. We know from the definition that the voltage is equal to the circulation of an electric field around a closed loop. (Faraday's law.)

It is convenient to rotate a great number N of loops in this magnet which creates a maximum emf of

(31.40) $$\varepsilon_{\max} = NBA\omega$$

The electric power created in this way is not free. It is created by the power supplied by the mechanical device rotating the coils in the magnetic field. We are dealing with a power or energy transformation.

An **electromotor** is a device in which **electric energy** is transformed into **mechanical work**. It functions very much like a generator running in reverse.
Instead of generating current by mechanically rotating a coil, a dc current is supplied to the coil by an outside electric power supply like a battery. The coil is situated in a magnetic field and acts as a magnetic dipole. The **torque acting on the current carrying coil** causes it to rotate

$$\vec{\tau} = \vec{\mu} \times \vec{B} = NI\vec{A} \times \vec{B}$$

$$\Rightarrow \tau = \mu B \sin\theta \quad .$$

Provisions need to be made in order not to reverse the direction of the torque every half cycle. (Commutators, brushes, electronic timers.)
The motor is designed to produce a continuing rotation of the coils, which can then be used to do mechanical work.
However in a dc circuit a back emf is being created. As the coil rotates in the magnetic field, the changing magnetic flux induces a secondary or back-emf in the coil; this induced emf always acts to reduce the original current in the coil (Lenz's law). This is why it is called a back emf.
The back-emf increases in magnitude as the rotational speed of the coil increases $\varepsilon_{max} = NBA\omega$.
When a motor is activated, by pulling the trigger of a circular saw, for example, there is initially no back-emf; thus the original current is at its maximum $I = \frac{\Delta V}{R}$, determined by the resitance in the circuit (mostly the coil wiring, e.g. around 10 Ohms.) This situation must only be allowed to go on for a few seconds so that no overheating occurs. As the coil begins to rotate, the induced emf (e.g. 110V) opposes the outside applied voltage of 120V, which therefore decreases, causing the current to decrease as well $I = \frac{120V - 110V}{10\Omega}$. If the mechanical load increases (the circular saw cuts through a tough piece of wood), the rotational speed slows down, which causes the back-emf to decrease, to e.g.100V. This reduction in the back-emf increases the net voltage in the circuit and therefore the current in the coil $I = \frac{120V - 100V}{10\Omega} = 1.2A$, thus drawing more power RI^2 from the external power supply. For this reason, the power requirements for starting a motor are larger than for normal operation, and they are larger for running it with a greater load rather than with a smaller one.

If the motor is allowed to run under no mechanical load, the back emf must be sufficiently large so that the current in the coil does not cause overheating and burn-out of the motor. This leads us to consider the obvious danger of a jammed motor. If the motor jams, the rotation stops, and no more emf is reducing the applied voltage. Thus the internal current is given by the outside voltage, say 120V, divided by the resistance of maybe 10 Ohms. We will have a current of 12A, instead of 1 A. The motor will burn up.

This **back-emf** is illustrated in the following problem:

An electro motor in normal operation carries a direct current of **0.850A** when connected to a **120 V** power supply. The resistance of the motor windings is **11.8 Ohms**.
a)While in normal operation, what is the **back emf** generated in the motor?
IR =10 Volts is the voltage in the circuit. Therefore the difference between the voltage delivered to the circuit and IR must be the back emf, which means it is 110Volts.
$120V - RI - \varepsilon = 0$
b) At what rate is internal energy produced in the windings?
Internal energy means heat energy, which is produced in the resistor.
$Power = RI^2 = 8.53W$
c)Suppose that a malfunction stops the motor shaft (in the coil) from rotating. At what rate will internal energy be produced in that case?
The emf being 0, the current will be $I = 120V/11.8\Omega = 10.2A$
$Power = RI^2 = 1.22kW = I\Delta V$

31.7 Maxwell's equations.

(31.41)
Gauss: $div\vec{E} = \vec{\nabla}\bullet\vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow$ integral form: $\oiint_{\partial V} \vec{E}\bullet d\vec{A} = \Phi_E = \frac{Q}{\varepsilon_0}$
The flux of the electrostatic field $\vec{E}$ through a closed surface is equal to the charge contained within the volume of this surface, divided by ε_0.

Add to this:

(31.42)
$$\vec{E} = -\overrightarrow{grad}V \equiv -\vec{\nabla}\cdot V \Rightarrow \Delta V = -\frac{\rho}{\varepsilon_0} \Rightarrow V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$$
$$\vec{\nabla}\times\vec{E} = \vec{0} \text{ conservative field}$$

(31.43)
$$\text{Electric dipole: } \vec{p} = q\vec{d};\ \vec{\tau} = \vec{p}\times\vec{E};\ U_E = -\vec{p}\cdot\vec{E}$$

(31.44)
Ampere-Maxwell: $curl\vec{B} \equiv \vec{\nabla}\times\vec{B} = \mu_0\vec{j}$
$$\oint_{\partial A} \vec{B}d\vec{s} = \iint_A \left(\mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)\bullet d\vec{A} = \mu_0 I + \mu_0 \underbrace{\varepsilon_0\frac{d\Phi_E}{dt}}_{I_d = displacement current}$$
Any current creates a magnetic field around it.
Any time changing electric flux creates a magnetic field around it.

Add to this the derivation of the law of Biot-Savart:

(31.45)

$\vec{A}$ is the vector potential:

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)=\mu_0\vec{j}=-\Delta\vec{A}$$

$$\vec{B}=\vec{\nabla}\times\vec{A}\Rightarrow\Delta\vec{A}=-\mu_0\vec{j}=-\mu_0 q\vec{v}\Rightarrow\vec{A}(r)=\frac{\mu_0}{4\pi}\frac{q\vec{v}}{r}$$

$$\Rightarrow\vec{B}=\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}\Rightarrow d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\vec{r}}{r^3}$$

(31.46) Lorentz force on a single moving: $\vec{F}=q\vec{E}+q\vec{v}\times\vec{B}$

(31.47) $$d\vec{F}=Id\vec{s}\times\vec{B}\Rightarrow\vec{F}=I\vec{L}\times\vec{B}$$

(31.48) Magnetic dipole: $\vec{\mu}=I\vec{A};\vec{\tau}=\vec{\mu}\times\vec{B};U_B=-\vec{\mu}\bullet\vec{B}$

Faraday: $curl\vec{E}_{ind}\equiv\vec{\nabla}\times\vec{E}_{ind}=-\frac{\partial\vec{B}}{\partial t}\Rightarrow$ integral form by Stokes law:

$$\underbrace{\oint_{\partial A}\vec{E}_{ind}\cdot d\vec{s}}_{emf}=-\frac{d}{dt}\iint_A\vec{B}\bullet d\vec{A}=-\frac{d\Phi_B}{dt}$$

The time change of the magnetic flux through an open surface is equal to an emf created in a conducting wire which is a closed boundary to the Ampereian surface.

$div\vec{B}\equiv\vec{\nabla}\bullet\vec{B}=0$ A magnetic field has no beginning and no end. There are no magnetic monopoles similar to positive and electric charges. The magnetic flux through a closed surface is always 0.

$$\oiint_A\vec{B}\bullet d\vec{A}=0$$