Chapter 30. Biot-Savart, Ampere's law, displacement current

(1.1) $\vec{F}_B = q\vec{v}\times\vec{B}$

(1.2) $d\vec{F}_B = I\left(d\vec{s}\times\vec{B}\right)$

(1.3) $\vec{\mu} = I\vec{A} =$ magnetic dipole moment

(1.4) $U_B = -\vec{\mu}\cdot\vec{B}\,;\, \vec{\tau}_B = \vec{\mu}\times\vec{B}$

(1.5) $curl\vec{B} = \mu_0\vec{j}$; permeability of space

$\iint_A curl\vec{B}\bullet d\vec{A} = \oint_{\partial A}\vec{B}d\vec{s} = \mu_0\iint_A \vec{j}\bullet d\vec{A} = \mu_0 I$ Ampere's law

(1.6) $d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\vec{u}_r}{r^2}$; Biot-Savart **;**

(1.7) $\vec{B} = \frac{\mu_0}{4\pi}\frac{q_1\vec{v}_1\times\vec{r}}{r^3}$

$\frac{\mu_0}{4\pi} = 10^{-7}; \mu_0 = 1.2566\cdot 10^{-6}$

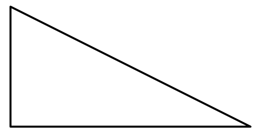
$\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I\cdot I'}{r}$ force between 2 parallel wires

(1.8) $curl\vec{B} = \mu_0\left(\vec{j} + \underbrace{\varepsilon_0\frac{\partial\vec{E}}{\partial t}}_{\vec{j}_d}\right)$ displacement current

1. In Niels Bohr's 1913 model of the hydrogen atom, an electron circles the proton at a distance of 0.529E-10 m with a speed of 2.19E6 m/s. Compute the magnitude of the magnetic field that this motion produces at the location of the proton. 12.5T
2. (3) a) A conductor in the shape of a square loop of edge length 0.400 m carries a current I =10.0 A in a clockwise direction. Calculate the magnitude and direction of the magnetic field at the center of the square. (28.3E-6T into the paper.)

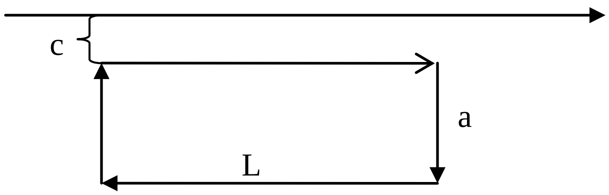
b) What if the conductor is bent into a single circular turn and carries the same current, what is the value of the magnetic field at the center? 24.7 E-6T.

3. A quarter circle of a wire is fed into on both ends by a straight wire, parallel to the radius. The wire carries a current of 5.00A. The radius of the arc is 3.00cm.Determine the magnitude and direction of the magnetic field at the origin. B=26.2E-6T.
4. (15) Two long parallel conductors carry currents of 3A into the page. Determine the direction and magnitude of the resultant B field at the perpendicular corner of a right triangle, whose hypotenuse (13cm) is the distance between the two wires, and which has sides 5 and 12 cm. 13E-6T, 113º



The magnetic force between two wires.

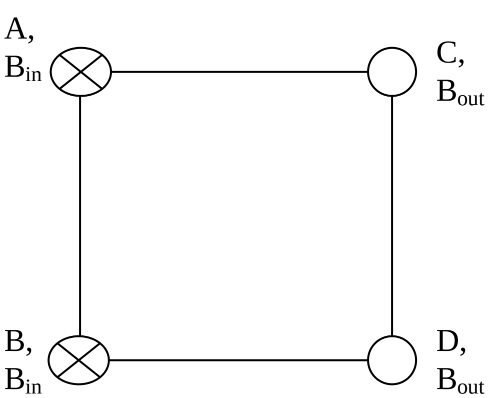
5. (17) In the following figure the current in the long straight wire is I=5A and the wire lies in the same plane as the rectangular loop below it. The rectangular loop carries a current of I'= 10A. The long side of the loop, parallel to current I, has a length of



0.45m, the small side has a length of 0.15m. The closest distance between the two wires is 0.1m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire. F=2.70E-5N upwards.

Ampere's law:

6. (23) Four long parallel conductors carry equal currents of 5A oriented perpendicular to the plane of the paper. The entry points into the paper form a square with its entry



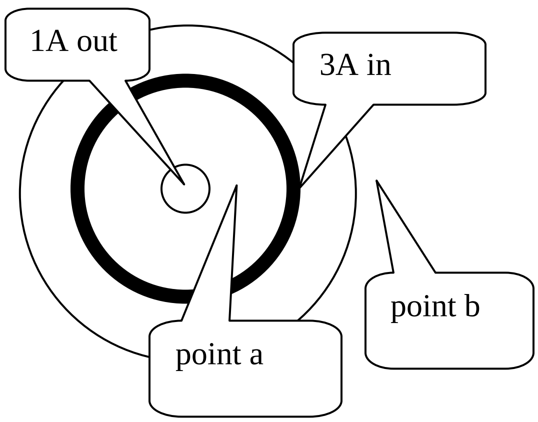
points into the page on the left named A (top) and B (bottom). This is the left side of the square. The right side of the square has the currents come out of the paper with points C on top and D at the bottom. The side length is 0.2m. Calculate the magnitude and direction of the magnetic field at the center of the square.
B=20E-6 T to the bottom of the page.

7. Calculate the circulation of the following vectorfield $\langle -5y, 3x, 0 \rangle$ around a semi circle with radius 5 using Stokes' theorem (Green's theorem). (100π)

8. (25) In a coaxial cable the center conductor is surrounded by a rubber layer, which is surrounded by an outer conductor, which is surrounded by another rubber layer. In a particular application, the current in the inner conductor is 1A out of the page and the

current in the outer conductor is 3A into the page. Determine the magnitude and direction of the magnetic field at a point a inside the inner rubber layer at 1mm from the center and at a point b outside the whole cable which lies 3 mm from the center of the cable.

a) 200E-6T toward top of page ccw, b) 133E-6 toward bottom of page cw.



9. (27) A packed bundle of 100 long, straight, insulated wires forms a cylinder of radius R=0.500cm. a) If each wire carries 2A, what are the magnitude and direction of the magnetic force per unit length acting on a wire located 0.200cm from the center of the bundle? (6.34E-3N/m inward.)

b) Would a wire on the outer edge of the bundle experience a force greater or smaller than the value calculated in part a). (The force is greatest at the outer surface.)

The magnetic field of a solenoid:

10. (35)What current is required in the windings of a long solenoid that has 1000 turns uniformly distributed over a length of 0.4m, to produce at the center of the solenoid a magnetic field of magnitude 1E-4T? (31.8mA)

Magnetic flux:

11. (39)A cube of edge length 2.5cm is positioned with its sides along the positive x-y-z coordinates. A uniform magnetic flux $\vec{B} = \langle 5,4,3 \rangle T$ exists throughout the region. Calculate the flux through the face in the y-z plane intersecting +x at 2.5cm. (3.12Wb)

What is the total flux through the six faces? (0)

Displacement current and the general form of Ampere's law.

12. A 0.1A current is charging a capacitor that has square plates 5cm on each side. The plate separation is 4mm. Find (a) the time rate of change of electric flux between the plates and (b) the displacement current between the plates. (a: 11.3E9 Vm/s; b:0.1A)

Magnetism in matter:

13. In Bohr's model of the hydrogen atom, the electron circles around the proton with v=2.19E6m/s with a radius of 0.529E-10 m.

a) What is the magnitude of the magnetic moment due to the electron's motion? (9.27E-24Am^2)

b) If the electron moves in a horizontal circle, counterclockwise as seen from above, what is the direction of the magnetic moment vector? (Downward.)

14. (53) Helmholtz coils. See lecture and lab.

$$\frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\vec{r}}{r^3}$$

15. Calculate the curl of $\vec{A}(r)=\frac{\mu_0}{4\pi}q_1\frac{\vec{v}_1}{r}=\frac{\mu_0}{4\pi}q_1\frac{\vec{v}_1}{\sqrt{x^2+y^2+z^2}}$

and confirm that

$$\vec{B}=curl\vec{A}=\frac{\mu_0}{4\pi}\frac{q_1\vec{v}_1\times\vec{r}}{r^3}$$