Electric Flux

1. An electric field with a magnitude of 3.50kN/C is applied along the x-axis. Calculate the electric flux through a rectangular plane 0.350 m wide and 0.700 m long assuming that a) the plane is parallel to the yz plane b) the plane is parallel to the xy-plane c) the plane contains the y-axis and its normal makes an angle of 40 degrees with the x-axis. (858Nm^2/C; 0; 657 Nm^2/C)
2. (1) A 40.0 cm diameter loop is rotated in an electric field until the maximum flux is found, which is 5.20E5Vm. Find the magnitude of the electric field. (4.14MN/C)
3. (5) A pyramid with horizontal square base, 6.00m on each side, and a height of 4 m is placed in a vertical electric field of 52.0 N/C. Calculate the total electric flux through the 4 slanted sides. (+1.87kNm^2/C)
4.

<u>Remember that total flux means "flux out" minus "flux in</u>"

Gauss's law:

5. Calculate $div\vec{E}$ and $curl\vec{E}$ for the following vector fields:

$$\langle x,y,z\rangle;\langle x^2y,yz^2,xyz\rangle;\frac{\langle x,y,z\rangle}{x^2+y^2+z^2{}^{\frac{3}{2}}}$$

6. Calculate the flux of the vector field $\langle x,y,z\rangle$ through the cylinder with radius a and height h. The central axis of the cylinder lies along the z-axis, the circular surface lies in the x-y plane, with z =0. Use Gauss' theorem

$$\oiint_{\partial V}\vec{E}\bullet d\vec{A}=\iiint_V div\vec{E}\cdot dV$$

a) find the volume integral
b) verify that the normal vector to the top and bottom of the cylinder is +and-$\vec{k}$.
c) verify that the flux through the top and bottom adds up to $h\iint_{top}dA=h\pi a^2$, (0 at the bottom).
d) verify that the normal unit vector in the horizontal direction is:

$$\frac{\langle x,y,0\rangle}{\sqrt{x^2+y^2}}=\frac{\vec{a}}{a}$$

It is constant along the cylindrical surface and the product with the vector $\vec{E}$ is $\vec{a}\frac{\vec{a}}{a}=a$

The flux through the sides of the cylinder is therefore: $a\iint_{side}dA=a\cdot 2\pi ah=2\pi a^2h$

Therefore, the total flux is equal to three times the volume of the cylinder.

7. Calculate the **total** electric flux through the circular cross section A of a paraboloid with circular cross-section if this cross section is perpendicular to an electric field E. What is the flux through the remaining surface of the paraboloid? ($E_0\pi r^2$)

8. (11) A point charge Q is located just above the flat surface of a hemisphere of radius R. What is the flux through the a) curved portion of the hemisphere?
   b) through the flat surface? $a)Q/2\varepsilon_0; b)-Q/2\varepsilon_0$

Application of Gauss' law to various charge distributions.

9. (19) Determine the magnitude of the electric field at the surface of a lead-208 nucleus which contains 82 protons and 126 neutrons. Assume that the lead nucleus has a volume 208 times the volume of a proton , which you may consider as a sphere of radius 1.20E-15m. (2.33E21N/C)
10. (23) A 10.0 g piece of Styrofoam carries a net charge of -0.700E-6 C. It floats above the center of a large horizontal sheet of plastic that has a uniform charge density on its surface. What is the charge per unit area on the plastic sheet? (-2.48E-6C/m^2)
11. (29) Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the center of the axis where r<R.($E=\dfrac{\rho r}{2\varepsilon_0}$ radially outward)
12. (29) Consider a thin spherical shell of radius 14.0 cm with a total charge of 32.0E-6C distributed uniformly on its surface. Find the electric field a) 10cm and b) 20cm from the center of the charge distribution. (a) 0; b) 7.19MN/C radially outward.)
13. (21) A large flat horizontal sheet of charge has a charge per unit area of 9.00E-6C/m^2. Find the electric field just above the middle of the sheet. (508kN/C)

Conductors in electrostatic equilibrium

14. (33) A long straight metal rod has a radius of 5.00cm and a linear charge $\lambda = 30nC/m$. Find the electric field a) 3.00cm b) 10.0 cm c) 100cm from the axis of the rod, where distances are measured perpendicularly to the rod.

a) 0; b) 5.4kN/C; c) 540 N/C

15. (39) A long straight wire is surrounded by a hollow metal cylinder of thickness d (for the metal), coaxial with the wire. The wire has a linear charge density λ and the cylinder has a net charge per unit length of 2λ.From this information, use Gauss' law to find a) the charge per unit length on the inner and outer surfaces of the cylinder and b) the electric field outside of the cylinder, a distance r from the axis. a)-λL and 3λL; b) $E=\dfrac{k_e 6\lambda}{r}$
16. A hollow conducting sphere is surrounded by a larger concentric spherical conducting shell of thickness d. The inner sphere has a net charge of –Q, and the outer shell has a net charge of +3Q. The charges are in electrostatic equilibrium. Using Gauss's law, find the charges and electric fields

everywhere. E=0 inside the sphere and the material of the shell,

$E = \frac{k_e Q}{r^2}$ between sphere and shell radially inward;

$E = \frac{2k_e Q}{r^2}$ outside the shell directed outward

-Q on the outer surface of the sphere

+Q on the inner surface of the shell

+2Q on the outer surface of the shell

Formal derivation of Gauss's law:

17. (43 modified) A sphere of radius R surrounds a point charge Q, located at its center. Using integration, calculate the surface portion of the sphere subtending a half-angle θ with the radius. Find the flux of the electric field through this area. A= $\frac{Q}{2\varepsilon_0}\left(1-\cos\theta\right)$

$$\Phi = \iint_A \vec{E}d\vec{A} = \frac{q}{4\pi\varepsilon_0}\frac{1}{R^2}2\pi R^2\left(1-\cos\theta\right) = \frac{q}{2\varepsilon_0}\left(1-\cos\theta\right)$$

18. (52) A sphere of radius 2a is made of a non-conducting material that has a uniform volume charge density ρ. (Assume that the material does not affect the electric field.) Place the center of this sphere at the origin of an xyz coordinate system, x to the right, y vertically up. A spherical cavity of radius a is now removed from the top part of the sphere, extending from y=0 to y=2a. The center of the cavity is now located at x=0, y=a, z=0. Show that the electric field within the cavity is uniform and is given by $E_x = 0$ and $E_y = \frac{\rho a}{3\varepsilon_0}$. Hint: The field within the cavity is the superposition of the field due to the original uncut sphere, plus the field due to a sphere the size of the cavity with a uniform negative charge density –ρ.