Ch2 Kinematics

When we describe the location and speed of a particle we do so in terms of space coordinates, x, y, z and a time coordinate t. We use these as mathematical representations of actual physical time and space. We usually carry over the mathematical assumptions to the physical assumptions. That is we consider the motion of an object like the motion of a **dimensionless point** which nevertheless has a mass m and possibly an electric charge +q or -q or some other physical attributes. Space and time coordinates x, y, z, t, and mass m are the mechanical variables in a mathematical sense and we assume that their values are taken from the set of real numbers $\mathbb{R}$

The location at the time t is described by a location vector (1.1) $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$.

We will deal with vectors and vector notation in a later chapter. Simply put, a vector describing the location of a particle combines all three coordinates into one ordered set of numbers.

The concept of a dimensionless point with a mass is a very useful abstraction, even though at first sight it may appear strange. But just think about this: If we would want to describe a baseball and its movement totally accurately we would need the size of the ball, its exact shape, and mass distribution, its deformation and surface properties and so on. Instead of going into these complicated things immediately, we start with the description of the center of mass of the baseball as it moves. The center of mass can be thought of at this stage of our understanding as the geometric center of the object. In the case of a spherical ball this is simply the center. Once we understand the concepts and laws describing the motion of the dimensionless point, we add many points together and come to the description of an **extended object**.

The location of a point at a particular time on a straight axis is then given by the coordinates x(t), y(t), z(t). We simply say "x of t" and so on.

Assume a point on an x axis, whose origin is at the 0 meter mark and the 0 seconds mark. (We use meters and seconds as our standard space and time measurements.) We draw the x axis vertically and the t axis horizontally. The location at time t_1 is $x_1(t_1)$ or simply x_1. The same for the location at time t_2.

The location difference (and any other finite difference) is defined by the Greek capital letter delta, or $\Delta x = x_2 - x_1 \text{ and } \Delta t = t_2 - t_1$. You see that $x_2 = x_1 + \Delta x \text{ and } t_2 = t_1 + \Delta t$

The **<u>average velocity</u>**, denoted by a horizontal bar on top of the letter, is defined as:

(1.2)
$$\overline{\mathrm{v}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{x_2(t_1 + \Delta t) - x_1(t_1)}{\Delta t} = \frac{\Delta x}{\Delta t}$$

We can rewrite this by using the rules of algebra as:

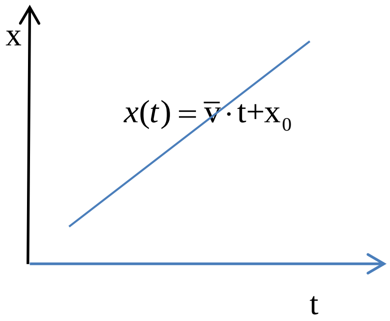
(1.3)
$$x_2 - x_1 = \overline{\mathrm{v}}(t_2 - t_1) \text{ or } x_2 = x_1 + \overline{\mathrm{v}} \cdot \Delta t$$

If we start with $t_1 = 0$ we get the equation for a straight line with the average velocity as the slope:

(1.4)
$$x_2 = x_1 + \overline{\mathrm{v}} \cdot t_2$$

Compare to y=mx+b

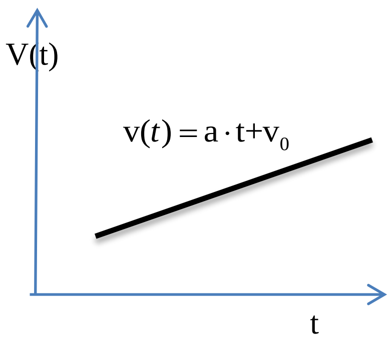
If we allow the time difference to get smaller and smaller we come to a value for the average velocity which approximates the instantaneous velocity in the x direction v(t). If we do this for all the points we come to the concept of instantaneous velocity at all time-points t. The velocity is then a continuous function of t, just like the location coordinate is a continuous function of t. The word "continuous" means that we have a value for t and x at any imaginable point on the x axis, no matter how small the time and space intervals are.



As the instantaneous velocity v(t) changes from moment to moment we can again define an average value of this change over time. This new is value is called average acceleration and is defined similarly to average velocity, namely:

(1.5) $$\bar{a}=\frac{\mathrm{v}_2-\mathrm{v}_1}{t_2-t_1}=\frac{\Delta \mathrm{v}}{\Delta t}$$

We again can define instantaneous acceleration just like in the case of instantaneous velocity. However, here in this chapter on kinematics we study the situation where the average acceleration is all there is, i.e. the instant acceleration is a constant.



This means that the increase in velocity over the same time interval is constant. In other words, the slope of the velocity function over time is constant and equal to the acceleration. In this situation of constant acceleration the average velocity between any two points is the velocity at the midpoint, i.e.:

(1.6) $$\overline{\mathrm{v}}=\frac{\mathrm{v}_1+\mathrm{v}_2}{2}$$

This is our second expression for average velocity. Let us set them equal to each other and find out what this means for the x value:

(1.7) $$\overline{\mathrm{v}}=\frac{\mathrm{v}_1+\mathrm{v}_2}{2}=\frac{\mathrm{x}_2-x_1}{t_2-t_1}$$

To simplify the algebra, we use $t_1 = 0 \textit{ and } v_1 = 0$.

(1.8) $$\frac{v_2}{2} = \frac{x_2 - x_1}{t_2} \Rightarrow x_2 - x_1 = \frac{v_2}{2} t_2 = \frac{a t_2}{2} t_2 = \frac{1}{2} a t_2^2$$

When we omit the index 2 we get the general equation:

(1.9) $$\boxed{x(t) = \frac{1}{2} a t^2 + v_0 t + x_0}$$

We rewrite the equation for velocity as:

(1.10) $$\boxed{v(t) = v_0 + at}$$

We can eliminate time t from the two equations above and arrive at the kinematic equation involving only velocities and locations, and constant acceleration.

(1.11) $$\boxed{v^2 = v_0^2 + 2a(x - x_0) \text{ both v and x are functions of time.}}$$

(1.12) $$\boxed{x = x_0 + \frac{v_0 + v}{2} t}$$