$$Springs:$$
$$y = A\cos\omega t;$$
$$\omega^2 = \frac{k}{m};$$
$$E = \frac{1}{2}kx^2 + \frac{1}{2}m\text{v}^2$$
$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$Simple\ pendulum:$$
$$\omega^2 = \frac{g}{l} \tag{0.1}$$

$$Waves:$$
$$y = A\sin(kx - \omega t); k = \frac{2\pi}{\lambda}; \omega = \frac{2\pi}{T}; \text{v} = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$$
$$transversal\ waves\ on\ strings: \text{v} = \sqrt{\frac{F_T}{m/L}} = \sqrt{\frac{F_T}{\mu}} \tag{0.2}$$
$$\text{sound-waves in water and other mediums with density } \rho: \text{v} = \sqrt{\frac{B}{\rho}}$$

Power and intensity of waves:

$$Waves\ on\ strings:$$
$$Power = \frac{1}{2}\mu\text{v}\left(\omega y_{\max}\right)^2$$

$$Sound\ waves:$$
$$I = \frac{1}{2}\rho\text{v}\left(\omega s_{\max}\right)^2; \rho = density\ of\ the\ medium\ (air, water, concrete, steel\ etc.)$$

Superposition (addition) of trigonometric functions:

$$\text{soundlevel } \beta = 10\log\frac{I}{I_0}$$

Interference of sine waves with the same amplitude but different arguments:

$$A_1\sin\theta_1 + A_1\sin\theta_2 = 2A_1\cos\frac{\theta_2 - \theta_1}{2}\sin\frac{\theta_2 + \theta_1}{2}$$

Double slit experiment:

If two waves differ only by a constant phase-shift they show the effect of destructive and constructive interference:

$$y_1 + y_2 = 2A\cos\frac{\phi}{2}\cdot\sin(kx-\omega t+\frac{\phi}{2})$$

(0.3) In the DSE the phaseshift is produced by different path lengths:

$$y_2 = A\sin[k\left(x+\Delta x\right)-\omega t];\ \Delta x = d\sin\theta$$

Constructive interference occurs when $\Delta x = n\lambda$

Standing Waves:

When waves occupy the same space superposition can form an interference pattern. Under certain conditions they form standing waves. Standing waves on a string under tension, for example occur on string instruments guitars, violins, cellos, etc. Two waves traveling in opposite directions interfere according to:

$$A\sin(kx-\omega t)+A\sin(kx+\omega t) = 2A\cos\left(\omega t\right)\sin kx$$

(0.4) If the string has length L and is tied down at both ends, we must have a 0 value for x=L

$$\sin kL = 0 \Rightarrow kL = n\pi \Rightarrow \lambda_n = \frac{2L}{n};\ f_n = \frac{v}{2L}n;\ n \text{ even and odd integers.}$$

If one end is closed and the other end open, the open end at x=L requires that:

$$\sin(kL) = \frac{2n+1}{2}\pi;\ n = 0,1,2,3$$

$$\frac{2\pi}{\lambda_{2n+1}}L = \frac{2n+1}{2}\pi \Rightarrow \lambda_{2n+1} = \frac{4L}{2n+1}$$

(0.5)

$$f_{2n+1} = \frac{v}{4L}\left(2n+1\right) \Rightarrow f_1 = \frac{v}{4L};\ f_3 = 3f_1;\ f_5 = 5f_1 \text{ the even overtones are missing.}$$

$$\text{example: } v = 343\frac{m}{s};\ L = 0.2meters \Rightarrow f_1 = \frac{343\frac{m}{s}}{0.2m} = 1715Hz;\ \lambda_1 = 0.8m;\ \lambda_3 = \frac{0.8}{3}m$$

Doppler effect: source moves, receiver is stationary;

Doppler effect for sound, source moves, observer is stationary:

$a)\lambda' = \lambda\left(1-\frac{s}{\mathrm{v}}\right)$ source approaching with speed s

(0.6) $b)\lambda' = \lambda\left(1+\frac{s}{\mathrm{v}}\right)$ source receding with speed s

To get from wavelengths λ to frequencies f, we just remember the general relationship $\lambda f = \mathrm{v}$ and $\lambda' f' = \mathrm{v};$ where v is the speed of the wave.

Doppler effect: receiver moves, source is stationary

The relationship between speed of sound and wavelength is in the reference frame of the source:

$\mathrm{v}=\lambda f$ or a) $\mathrm{f}=\frac{\mathrm{v}}{\lambda}$

In the reference frame of the moving receiver, it is

b)$\mathrm{v}'=\lambda f'$ or a) $\mathrm{f}'=\frac{\mathrm{v}'}{\lambda} \Rightarrow \frac{f'}{f} = \frac{\mathrm{v}'}{\mathrm{v}}$

As we have $\mathrm{v}'=\mathrm{v}\pm \mathrm{V_R}$, we get:

$c) f' = f\left(\frac{\mathrm{v}\pm \mathrm{V_R}}{\mathrm{v}}\right) = f\left(1\pm\frac{\mathrm{V_R}}{\mathrm{v}}\right)$

The frequency increases when the receiver moves towards the stationary source.

Relativistic Doppler effect for light:

(0.7) Doppler effect for light (relativistic):

$a)\lambda' = \lambda\sqrt{\frac{1-\frac{\mathrm{v}}{c}}{1+\frac{\mathrm{v}}{c}}};\ f' = f\sqrt{\frac{1+\frac{\mathrm{v}}{c}}{1-\frac{\mathrm{v}}{c}}}$ $\qquad b)\lambda' = \lambda\sqrt{\frac{1+\frac{\mathrm{v}}{c}}{1-\frac{\mathrm{v}}{c}}};\ f' = f\sqrt{\frac{1-\frac{\mathrm{v}}{c}}{1+\frac{\mathrm{v}}{c}}}$

a)for a star approaching with the speed v; blue shift
b)for a star receding with the speed v; red shift

Hubble constant: Once we know the speed of a galaxy we can approximate the distance R of the galaxy to our own with **Hubble's** formula:

$$R = \frac{v}{H}$$
$$H = \frac{0.017m}{s \cdot lightyears} \tag{0.8}$$

$$R = \frac{v}{H}$$
$$H = \frac{0.017m}{s \cdot lightyears} \tag{0.9}$$