Formulas for midterm 2:

1.

The work done by friction is equal to the change in potential energy + the change in kinetic energy.

$-f_k x = \Delta PE + \Delta KE = \Delta U + \Delta K$

Conservation of mechanical energy in the absence of frictional forces

E=K+U=constant; $+\Delta U = 0; K_i + U_i = K_f + U_f$

The work done on an object with mass m by all exterior forces:

$$W = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

2.

$$Power = \frac{dW}{dt} = \vec{F} \cdot \vec{v} \text{ in Watts}$$

3.

4. If there is a frictional force and N conservative forces, we have:

$$-f_k x = \sum_{l=1}^{N} \Delta U_l + \Delta K$$

$F = -kx$; restoring force of a spring; $U = \frac{1}{2} k x^2$

5. $\vec{F} = -mg\vec{j}$ force of gravity on the surface of the earth: U=mgy

6.

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = m\vec{a} + \frac{\Delta m}{\Delta t} \vec{v}$$

$1eV = 1.60 \cdot 10^{-16} J$

7. Impulse $\vec{J} = \vec{F} \cdot \Delta t = \Delta \vec{p}$

Momentum conservation: If all forces between objects are internal, momentum of the whole system of masses is conserved. Collisions are typical examples:

8. $$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

In perfectly elastic collisions the kinetic energy of the system is also conserved:

9. $$\underbrace{\frac{1}{2}m_1\vec{v}_{1i}^2 + \frac{1}{2}m_2\vec{v}_{2i}^2}_{\text{before collision}} = \underbrace{\frac{1}{2}m_1\vec{v}_{1f}^2 + \frac{1}{2}m_2\vec{v}_{2f}^2}_{\text{after collision}}$$

In perfectly elastic head-on collisions between two particles, we have:

10. $$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

An inelastic collision is typically a collision in which the objects stick together after colliding.. Kinetic energy is not conserved.

11. $$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

Rotational formulas

For the rotation of point masses around a fixed axis, with constant angular acceleration:

12. $$\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0; a_\theta = R\alpha$$
$$\omega = \alpha t + \omega_0; v = R\omega \Rightarrow \vec{v} = \vec{\omega} \times \vec{R}$$

13. $$\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} = I\alpha\vec{u}_z$$
$$\tau = I_A\alpha$$

14. The angular momentum **L** of a point mass m, moving with velocity **v** is given by the **vector product** between its radius vector **r** and the linear momentum vector **p.**

$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$; Note that the cross product between two parallel vectors is 0

If there are no exterior torques acting on a system, the angular momentum of the system is conserved. $\sum_{k=1}^{N} \vec{L}_{k\ initial} = \sum_{k=1}^{N} \vec{L}_{k\ final} \Rightarrow$

for a system of two rotating objects:

$$I_{1\,initial}\omega_{1\,initial} + I_{2\,initial}\omega_{2\,initial} = I_{1\ final}\omega_{1\ final} + I_{2\ final}\omega_{2\ final}$$

15.

The change of total momentum of a system is equal to the sum of the EXTERIOR TORQUES:

$$\frac{\Delta \vec{L}}{\Delta t} = \vec{\tau}_{exterior}$$

16. $L = mr^2\omega$

Moment of inertia:

17.

$$I = \sum_{i=1}^{N} m_i r_i^2$$

$$Solid\ Cylinder:\ \mathrm{I}_{cm} = \frac{1}{2}MR^2;\ \text{solid sphere: } \mathrm{I}_{cm} = \frac{2}{5}MR^2;\ Linear\ rod: \mathrm{I}_{cm} = \frac{1}{12}ML^2$$

Kinetic energy of rotation:

$$K = \frac{1}{2} I_A \omega^2;$$

Parallel Axes Theorem:

$$I_A = I_{cm} + Md^2$$

18. (d is the distance between the two axes of rotation.)

Angular momentum of a solid object rotating around a fixed axis:

19. $L = I_A \omega$

Static equilibrium:

A system of masses is in static equilibrium if the sum of the exterior forces is 0 and the sum of the exterior torques is 0.

$$\rho_{air} = 1.29 \frac{kg}{m^3}; \rho_{He} = 0.179 \frac{kg}{m^3}; \rho_{water} = 1000 \frac{kg}{m^3} = 1.00 \frac{g}{cm^3}$$

20. Pascals principle:

The pressure inside of a liquid is the same everywhere at the same depth of the liquid.

21. Hydraulic press:

Pressure on small area equals pressure on large area:

22. $F_{in} A_{in} = F_{out} A_{out}$
23. The buoyancy force pushing upwards on an object immersed in a liquid is equal to the weight of the displaced liquid.
24. $B = \rho_{liquid} g V$

25. For a floating object this means:

$$\rho_{objext} V_{object} = \rho_{liquid} V_{liquid}$$

Continuity equation:

26. $A_1 \mathrm{v}_1 = A_2 \mathrm{v}_2$

Bernoulli's law:

27. $P + \frac{1}{2} \rho \mathrm{v}^2 + \rho g y = \text{constant}$